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University of Chinese Academy of Sciences

卫星摄动方程

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2019年秋季

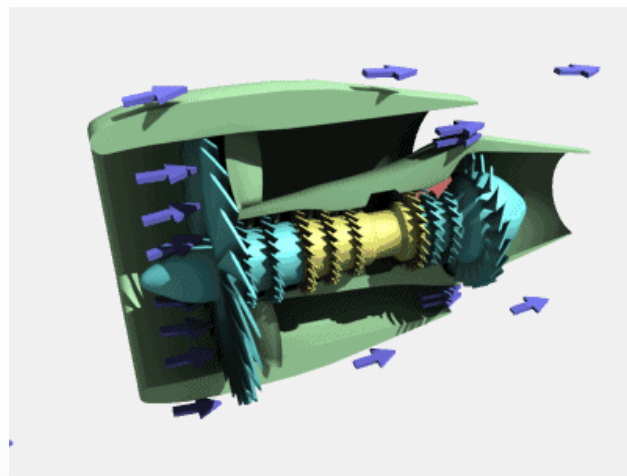
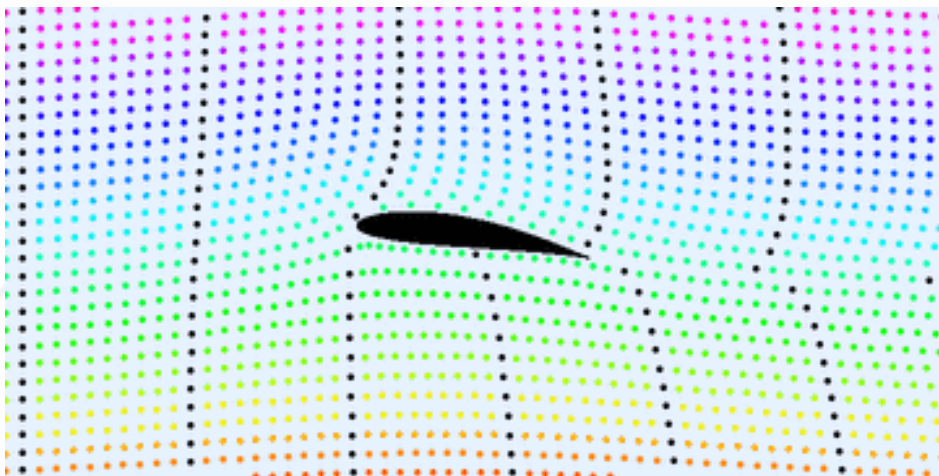
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卫星摄动方程

- 卫星受力情况
- 摄动方程的建立
- 一般摄动法简介
- 半解析法
- 摄动法应用

航空器与航天器飞行原理的区别



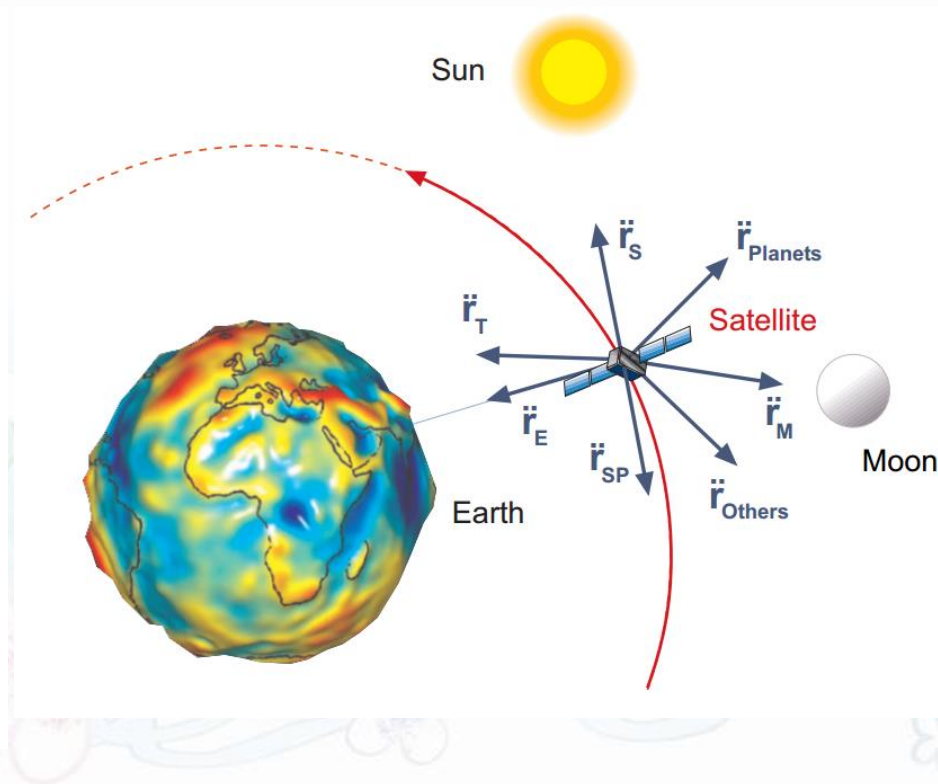
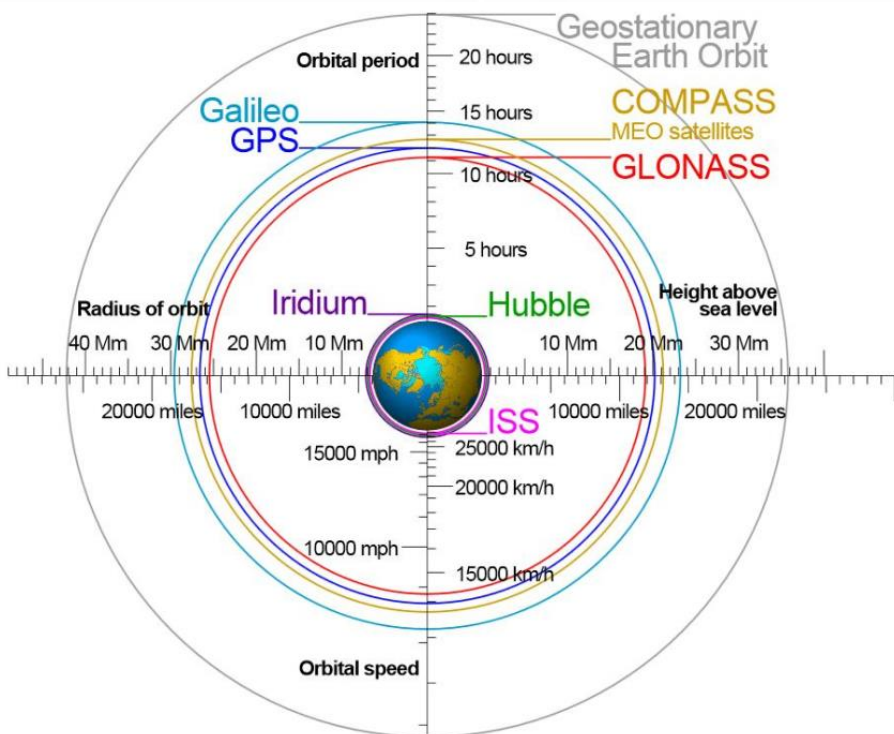
伯努利原理：动能+重力势能+压力势能=常数

飞机飞行时机翼周围空气的流线分布是指机翼横截面的形状上下不对称,机翼上方的流线密,流速大,下方的流线疏,流速小。由伯努利方程可知,机翼上方的压强小,下方的压强大。这样就产生了作用在机翼上的方向的升力。

航天器飞行原理则满足基本的天体力学规律



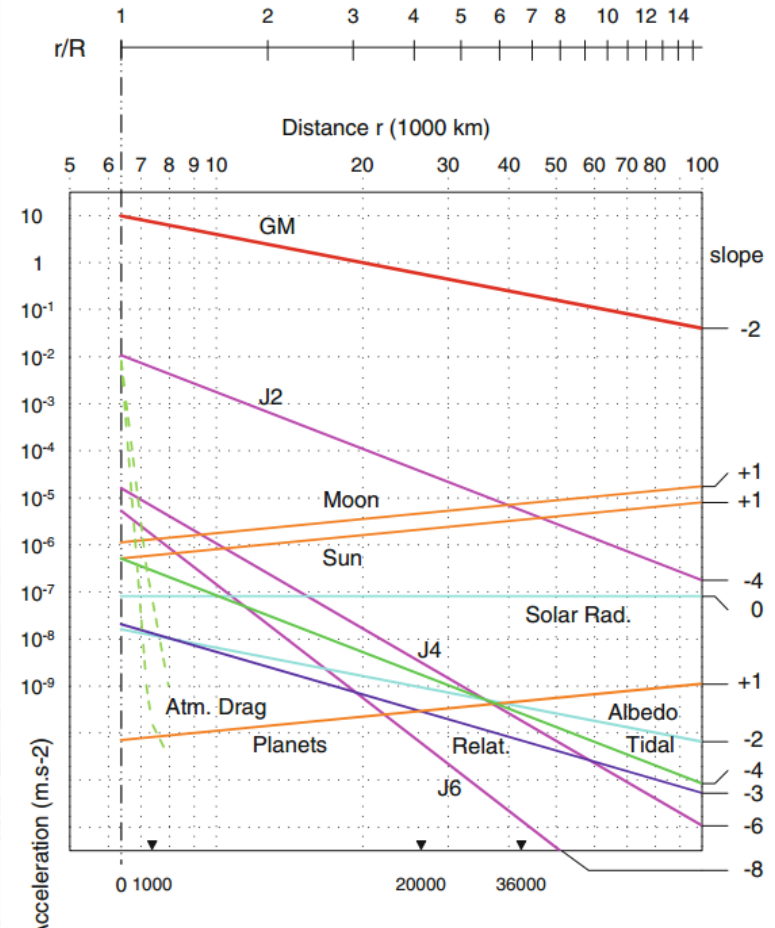
真实的卫星运动



- $h = 1,000$ km for satellites in low orbit (LEO),
- $h = 20,000$ km for positioning satellites (MEO),
- $h = 36,000$ km for geostationary satellites (GEO).

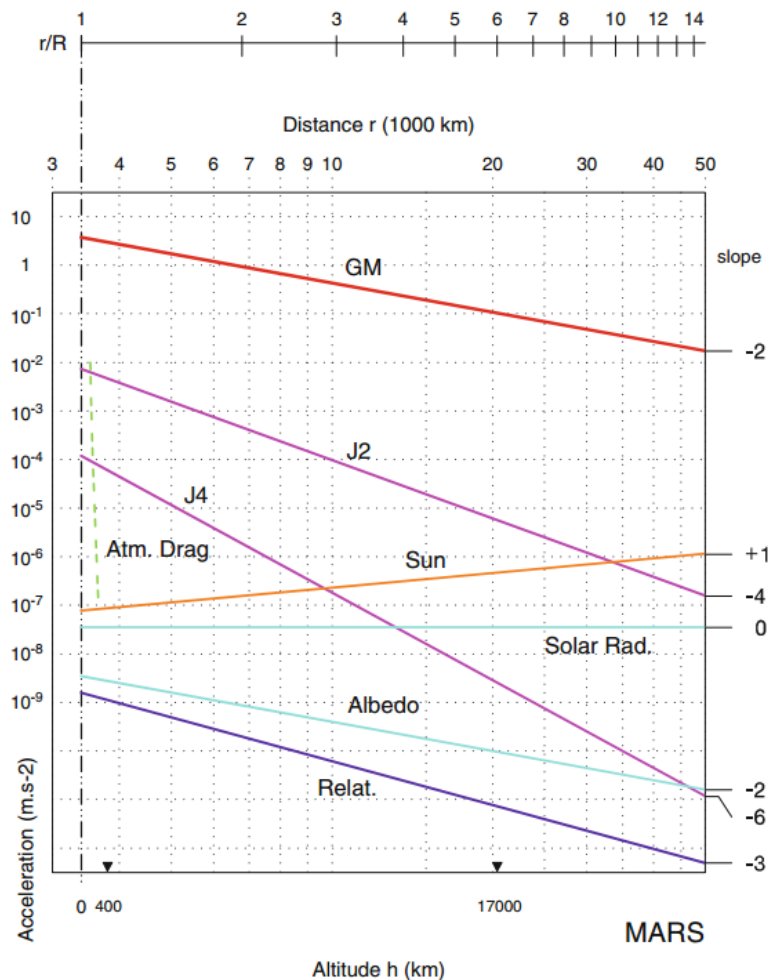
地球卫星摄动力量级

Symbol		Type of force
C		Conservative forces
	CC	<ul style="list-style-type: none"> ● Attraction of the Earth ● ○ Central term $\mu = GM$ ● ○ Other terms than CCC
	CCC	
	CCN	
	CL	<ul style="list-style-type: none"> ● Attraction of the Moon ● Attraction of the Sun
	CS	
	CP	<ul style="list-style-type: none"> ● Attraction by other planets ● Tidal effects (land, oceans) ● Relativistic effects
	CT	
CR		
D		Dissipative forces
	DF	<ul style="list-style-type: none"> ● Atmospheric drag ● Solar radiation pressure ● Albedo effect
	DP	
	DA	



- 10^{-3} for the perturbation due to the flattening of the Earth,
- 10^{-6} for perturbations due to other irregularities of the geoid.
- 10^{-7} for the attraction of the Moon,
- 10^{-8} for the attraction of the Sun.

火星卫星摄动量级

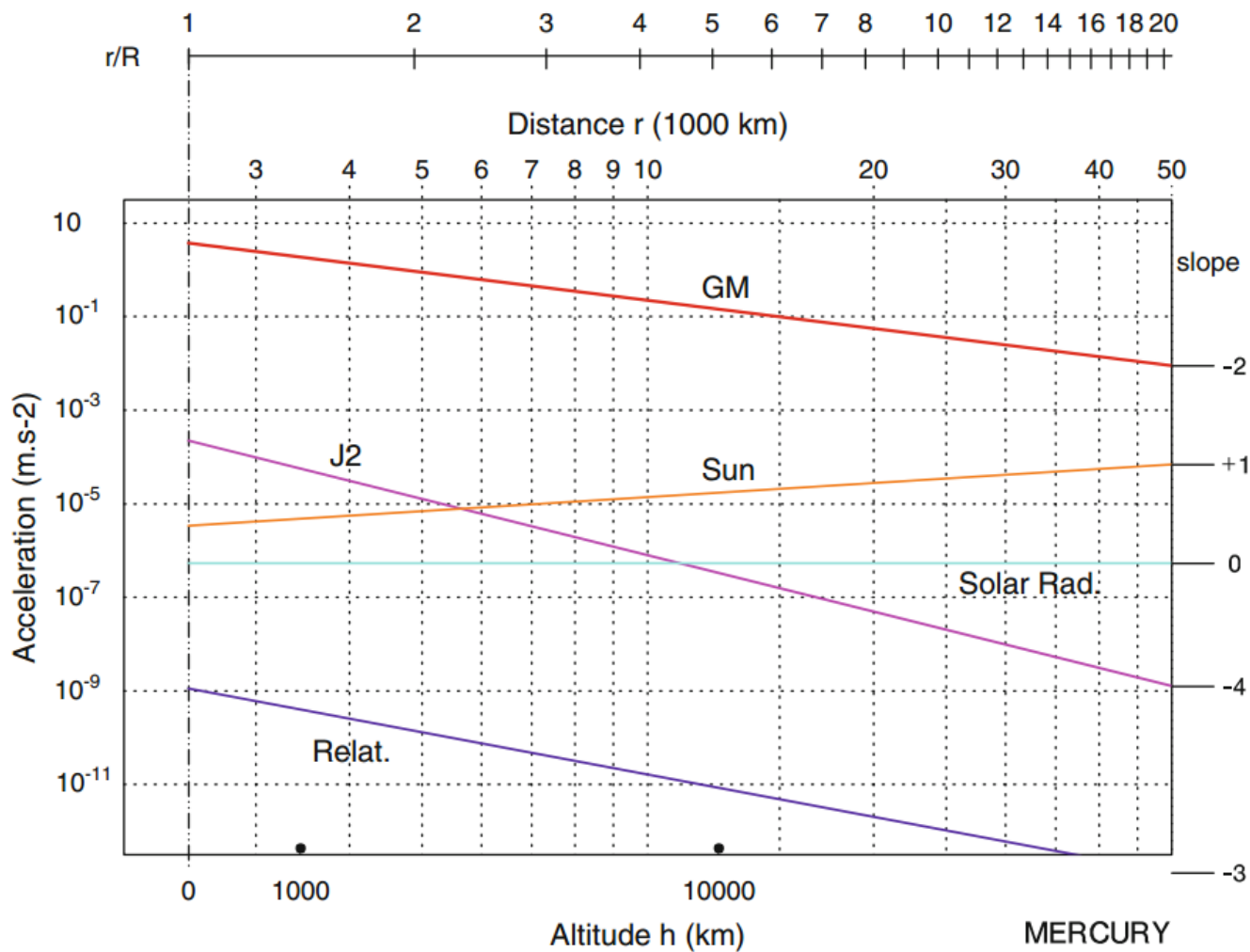


Quantity	Symbol	Unit	Mars	Earth
Gravitational constant	$\mu = GM$	$\text{m}^3 \text{s}^{-2}$	4.2828369 10^{13}	3.9860044 10^{14}
Equatorial radius	R	km	3,397.000	6,378.137
Flattening	$1/f$	-	154.40915	298.25766
Equatorial circumference	$L_{P/\text{equat}}$	km	21,343.980	40,075.012
Meridian circumference	L_M	km	21,274.922	40,007.860
Acceleration (equatorial)	g	m s^{-2}	3.7052	9.7803
Acceleration (45°)	g	m s^{-2}	3.7214	9.8061
Acceleration (pole)	g	m s^{-2}	3.7376	9.8321
Gravitational potential	$J_2 \times 10^6$	-	1,955.4513	1,082.6267
Gravitational potential	$J_3 \times 10^6$	-	+31.4559	-2.5327
Gravitational potential	$J_4 \times 10^6$	-	-15.3694	-1.6196
Ratio (frozen orbit)	$J_3/J_2 \times 10^3$	-	+16.0863	-2.3394
Semi-major axis	a	a.u.	1.52366	1.00000
Period of revolution				
sidereal	N_{sid}	day	686.9800	365.2564
tropical	N_{tro}	day	686.9725	365.2422
anomalistic	N_{ano}	day	686.9951	365.2496
Angular speed	$\Omega_T \times 10^5$	rad s^{-1}	7.088218	7.292115
	Ω_T	$^\circ \text{d}^{-1}$	350.891983	360.985559
Period of rotation				
sidereal	D_{sid}	h	24.622962	23.934471
		s	88,642.663	86,164.090
mean solar day	D_M	h	24.6598	24.0000
		s	88,775.245	86,400.000
Inclination/ecliptic	i	deg	1.8496	-
Obliquity	ϵ	deg	25.19	23.44
Eccentricity	e	-	0.0930	0.01671

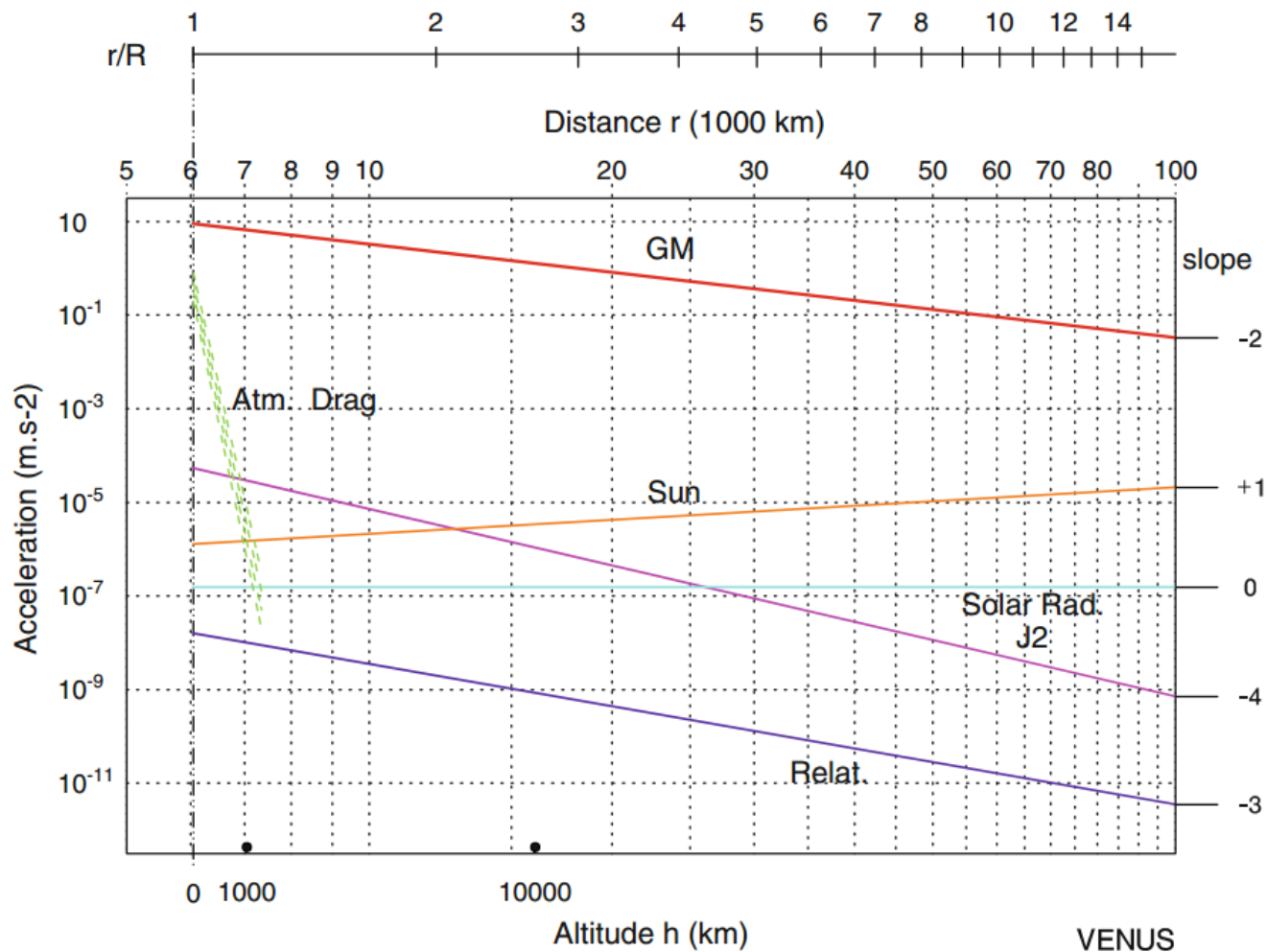
Central and perturbative accelerations as a function of the distance r of the satellite from the center of Mars. Log-log scale. In the range of variation considered, the curves are approximately straight lines and their slope is indicated. The altitudes of two types of satellite have also been noted.

Geodetic and astronomical data for Mars and the Earth. For the units, a dash means dimensionless.

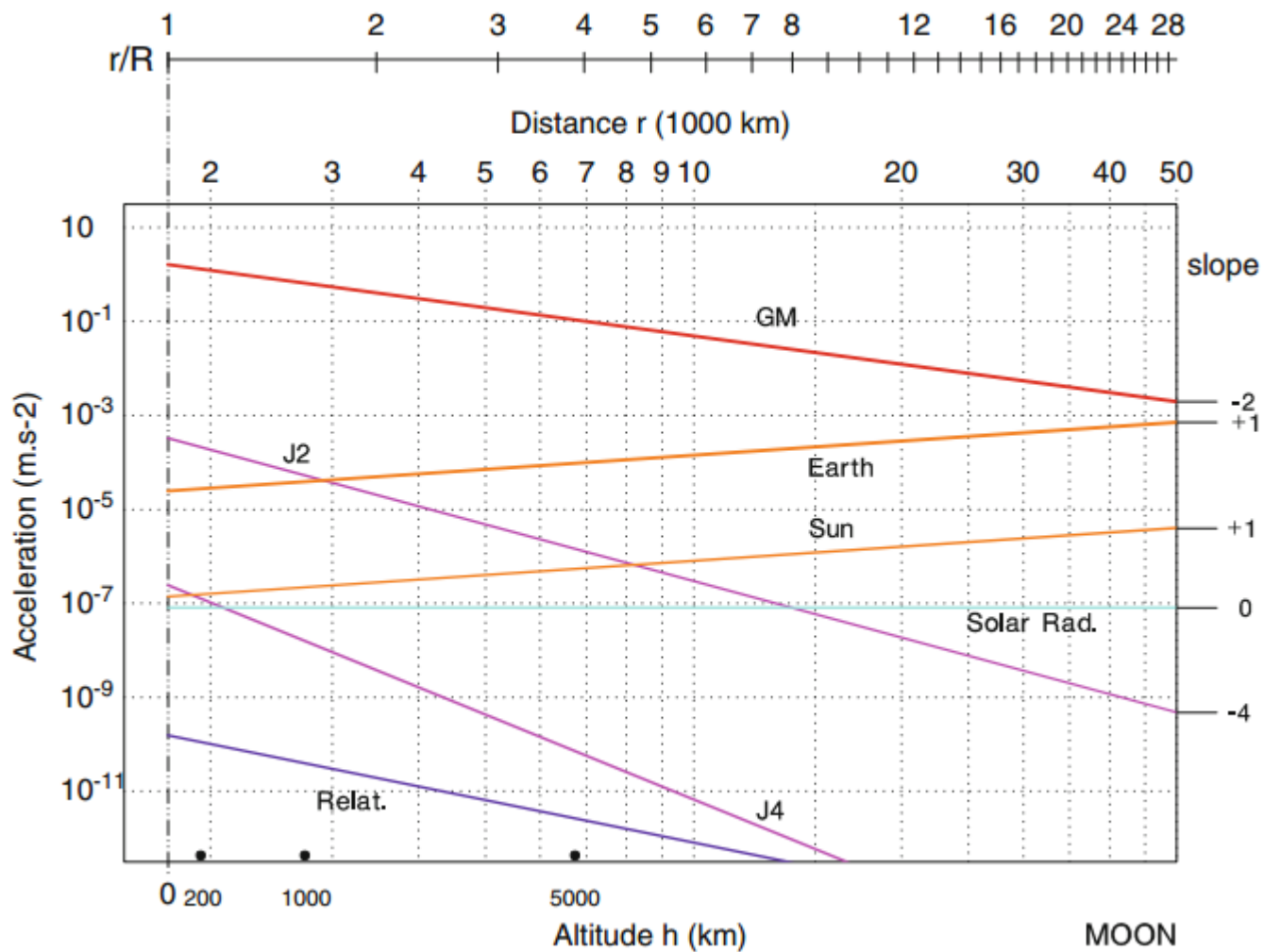
水星卫星摄动量级



金星卫星摄动量级



月球卫星摄动量级



引力加速度

$$\ddot{\mathbf{r}} = \mathbf{grad} U, \quad U = U_0 + \mathcal{R}$$

$$\frac{d^2x}{dt^2} = \frac{\partial U}{\partial x}, \quad \frac{d^2y}{dt^2} = \frac{\partial U}{\partial y}, \quad \frac{d^2z}{dt^2} = \frac{\partial U}{\partial z}$$

$$\dot{x} = \frac{dx}{dt}$$
$$\frac{d\dot{x}}{dt} = \frac{\partial U}{\partial x}$$

拉格朗日括号

记 $\xi_1 = x, \quad \xi_2 = y, \quad \xi_3 = z$

$$\ddot{\xi}_i = \frac{\partial U}{\partial \xi_i}$$

记 $S_1 = a, \quad S_2 = e, \quad S_3 = i, \quad S_4 = \Omega, \quad S_5 = \omega, \quad S_6 = \Theta$

$$\Theta = M - nt = -nt_p$$

运动方程可以写为

$$\sum_{j=1}^6 \frac{\partial \xi_i}{\partial S_j} \frac{dS_j}{dt} = \dot{\xi}_i, \quad i = 1, 2, 3,$$

$$\sum_{j=1}^6 \frac{\partial \dot{\xi}_i}{\partial S_j} \frac{dS_j}{dt} = \frac{\partial U}{\partial \xi_i}, \quad i = 1, 2, 3$$

拉格朗日括号

Multiplying the three equations by $-\partial\dot{\xi}_i/\partial S_k$ and the three equations by $\partial\xi_i/\partial S_k$ and adding, we obtain the system

$$\sum_{i=1}^3 \sum_{j=1}^6 \left(-\frac{\partial\dot{\xi}_i}{\partial S_k} \frac{\partial\xi_i}{\partial S_j} + \frac{\partial\xi_i}{\partial S_k} \frac{\partial\dot{\xi}_i}{\partial S_j} \right) \frac{dS_j}{dt} = \sum_{i=1}^3 \left(-\frac{\partial\dot{\xi}_i}{\partial S_k} \dot{\xi}_i + \frac{\partial\xi_i}{\partial S_k} \frac{\partial U}{\partial \xi_i} \right)$$

$$k = 1, 2, 3, 4, 5, 6.$$

使用如下记号

$$[S_k; S_j] = \sum_{i=1}^3 \left(\frac{\partial\xi_i}{\partial S_k} \frac{\partial\dot{\xi}_i}{\partial S_j} - \frac{\partial\dot{\xi}_i}{\partial S_k} \frac{\partial\xi_i}{\partial S_j} \right)$$

拉格朗日括号

拉格朗日括号表示的运动方程

$$-\sum_{i=1}^3 \frac{\partial \dot{\xi}_i}{\partial S_k} \dot{\xi}_i = -\frac{1}{2} \sum_{i=1}^3 \frac{\partial \dot{\xi}_i^2}{\partial S_k} = -\frac{1}{2} \frac{\partial}{\partial S_k} \sum_{i=1}^3 \dot{\xi}_i^2$$

$$T = \frac{1}{2} \sum_{i=1}^3 \dot{\xi}_i^2 = \frac{1}{2} V^2$$

$$\sum_{i=1}^3 \frac{\partial U}{\partial \xi_i} \frac{\partial \xi_i}{\partial S_k} = \frac{\partial U}{\partial S_k}$$

若记

$$F = U - T$$

则运动方程可以写为

F与哈密顿函数反号

$$\sum_{j=1}^6 [S_k; S_j] \frac{dS_j}{dt} = \frac{\partial F}{\partial S_k}, \quad k = 1, 2, 3, 4, 5, 6$$

轨道面内坐标的根数表示

$$X = r \cos v = a(\cos E - e),$$

$$Y = r \sin v = a\sqrt{1 - e^2} \sin E,$$

$$Z = 0.$$

$$(1 - e \cos E) \frac{de}{dt} = n = \sqrt{\frac{\mu}{a^3}}$$

$$\dot{X} = -\sqrt{\frac{\mu}{a}} \frac{\sin E}{1 - e \cos E},$$

$$\dot{Y} = \sqrt{\frac{\mu}{a}} \sqrt{1 - e^2} \frac{\cos E}{1 - e \cos E},$$

$$\dot{Z} = 0.$$

空间坐标的根数表示

$$P = \begin{pmatrix} \cos \Omega \cos \omega & -\cos \Omega \sin \omega & \sin \Omega \sin i \\ -\sin \Omega \sin \omega \cos i & -\sin \Omega \cos \omega \cos i & \\ \sin \Omega \cos \omega & -\sin \Omega \sin \omega & -\cos \Omega \sin i \\ +\cos \Omega \sin \omega \cos i & +\cos \Omega \cos \omega \cos i & \\ \sin \omega \sin i & \cos \omega \sin i & \cos i \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = P(\Omega, i, \omega) \begin{pmatrix} X(a, e, E) \\ Y(a, e, E) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = P(\Omega, i, \omega) \begin{pmatrix} \dot{X}(a, e, E) \\ \dot{Y}(a, e, E) \\ 0 \end{pmatrix}$$

$$\xi_i = P_{i1}X + P_{i2}Y, \quad \dot{\xi}_i = P_{i1}\dot{X} + P_{i2}\dot{Y}, \quad i = 1, 2, 3$$

坐标关于根数的偏导数

$$\frac{\partial \xi_i}{\partial S_k} = \frac{\partial P_{i1}}{\partial S_k} X + \frac{\partial P_{i2}}{\partial S_k} Y \quad \text{if } S_k = \Omega, i, \omega ,$$
$$\frac{\partial \xi_i}{\partial S_k} = P_{i1} \frac{\partial X}{\partial S_k} + P_{i2} \frac{\partial Y}{\partial S_k} \quad \text{if } S_k = a, e, E ,$$

$$\frac{\partial \dot{\xi}_i}{\partial S_k} = \frac{\partial P_{i1}}{\partial S_k} \dot{X} + \frac{\partial P_{i2}}{\partial S_k} \dot{Y} \quad \text{if } S_k = \Omega, i, \omega ,$$
$$\frac{\partial \dot{\xi}_i}{\partial S_k} = P_{i1} \frac{\partial \dot{X}}{\partial S_k} + P_{i2} \frac{\partial \dot{Y}}{\partial S_k} \quad \text{if } S_k = a, e, E .$$

坐标关于根数的偏导数

$$\frac{\partial X}{\partial a} = \cos E - e, \quad \frac{\partial X}{\partial e} = -a, \quad \frac{\partial X}{\partial M} = -a \sin E \frac{1}{1 - \cos E}$$

$$\frac{\partial Y}{\partial a} = 0, \quad \frac{\partial Y}{\partial e} = 0, \quad \frac{\partial Y}{\partial M} = a \sqrt{\frac{1+e}{1-e}}$$

$$\frac{\partial \dot{X}}{\partial a} = 0, \quad \frac{\partial \dot{X}}{\partial e} = 0, \quad \frac{\partial \dot{X}}{\partial M} = -\frac{na}{(1-e)^2},$$

$$\frac{\partial \dot{Y}}{\partial a} = -\frac{n}{2} \sqrt{\frac{1+e}{1-e}}, \quad \frac{\partial \dot{Y}}{\partial e} = \frac{na}{(1-e)^{3/2}}, \quad \frac{\partial \dot{Y}}{\partial M} = 0$$

计算[a;M]

$$[a; M] = \sum_{i=1}^3 \left(-\frac{\partial \dot{\xi}_i}{\partial a} \frac{\partial \xi_i}{\partial M} + \frac{\partial \xi_i}{\partial a} \frac{\partial \dot{\xi}_i}{\partial M} \right)$$

$$\frac{\partial \xi_i}{\partial a} = P_{i1} \frac{\partial X}{\partial a} + P_{i2} \frac{\partial Y}{\partial a} = P_{i1}(1 - e),$$

$$\frac{\partial \dot{\xi}_i}{\partial a} = P_{i1} \frac{\partial \dot{X}}{\partial a} + P_{i2} \frac{\partial \dot{Y}}{\partial a} = -P_{i2} \frac{n}{2} \sqrt{\frac{1+e}{1-e}}$$

$$\frac{\partial \xi_i}{\partial M} = P_{i1} \frac{\partial X}{\partial M} + P_{i2} \frac{\partial Y}{\partial M} = P_{i2} a \sqrt{\frac{1+e}{1-e}},$$

$$\frac{\partial \dot{\xi}_i}{\partial M} = P_{i1} \frac{\partial \dot{X}}{\partial M} + P_{i2} \frac{\partial \dot{Y}}{\partial M} = -P_{i1} \frac{na}{(1-e)^2}$$

$$[a; M] = \frac{na}{1-e} \sum_{i=1}^3 \left(-P_{i1}^2 + \frac{1+e}{2} P_{i2}^2 \right)$$

$$[a; M] = \frac{na}{1-e} \left(-1 + \frac{1+e}{2} \right) = -\frac{na}{2}$$

其他拉格朗日括号

$$[a; \Omega] = -[\Omega; a] = -\frac{na}{2} \sqrt{1-e^2} \cos i$$

$$[a; \omega] = -[\omega; a] = -\frac{na}{2} \sqrt{1-e^2},$$

$$[a; M] = -[M; a] = -\frac{na}{2},$$

$$[e; \Omega] = -[\Omega; e] = +na^2 \frac{e}{\sqrt{1-e^2}} \cos i$$

$$[e; \omega] = -[\omega; e] = +na^2 \frac{e}{\sqrt{1-e^2}},$$

$$[i; \Omega] = -[\Omega; i] = +na^2 \sqrt{1-e^2} \sin i$$

势能减动能关于根数的偏导数

$$\frac{\partial F}{\partial a} = + [a; \Omega] \dot{\Omega} + [a; \omega] \dot{\omega} + [a; M] \dot{M}$$

$$\frac{\partial F}{\partial e} = + [e; \Omega] \dot{\Omega} + [e; \omega] \dot{\omega} ,$$

$$\frac{\partial F}{\partial i} = + [i; \Omega] \dot{\Omega} ,$$

$$\frac{\partial F}{\partial \Omega} = - [a; \Omega] \dot{a} - [e; \Omega] \dot{e} - [i; \Omega] \dot{i} ,$$

$$\frac{\partial F}{\partial \omega} = - [a; \omega] \dot{a} - [e; \omega] \dot{e} ,$$

$$\frac{\partial F}{\partial M} = - [a; M] \dot{a} .$$

拉格朗日方程

$$\begin{pmatrix} da/dt \\ de/dt \\ di/dt \\ d\Omega/dt \\ d\omega/dt \\ dM/dt - n \end{pmatrix} = L \begin{pmatrix} \partial\mathcal{R}/\partial a \\ \partial\mathcal{R}/\partial e \\ \partial\mathcal{R}/\partial i \\ \partial\mathcal{R}/\partial\Omega \\ \partial\mathcal{R}/\partial\omega \\ \partial\mathcal{R}/\partial M \end{pmatrix}$$

$$L = \frac{1}{na^2} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & +2a \\ 0 & 0 & 0 & 0 & -\tau\bar{\tau} & +\tau \\ 0 & 0 & 0 & -\sigma\bar{\tau} & +\sigma\bar{\tau} \cos i & 0 \\ 0 & 0 & +\sigma\tau & 0 & 0 & 0 \\ 0 & +\tau\bar{\tau} & -\sigma\bar{\tau} \cos i & 0 & 0 & 0 \\ -2a & -\tau & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\sigma = \frac{1}{\sin i}, \quad \tau = \frac{1 - e^2}{e}, \quad \bar{\tau} = \frac{1}{\sqrt{1 - e^2}}$$

拉格朗日方程

$$\frac{da}{dt} = \frac{1}{na} \left(2 \frac{\partial \mathcal{R}}{\partial M} \right)$$

$$\frac{de}{dt} = \frac{1}{na^2} \frac{1-e^2}{e} \left(-\frac{1}{\sqrt{1-e^2}} \frac{\partial \mathcal{R}}{\partial \omega} + \frac{\partial \mathcal{R}}{\partial M} \right)$$

$$\frac{di}{dt} = \frac{1}{na^2 \sqrt{1-e^2} \sin i} \left(-\frac{\partial \mathcal{R}}{\partial \Omega} + \cos i \frac{\partial \mathcal{R}}{\partial \omega} \right)$$

$$\frac{d\Omega}{dt} = \frac{1}{na^2 \sqrt{1-e^2} \sin i} \left(\frac{\partial \mathcal{R}}{\partial i} \right)$$

$$\frac{d\omega}{dt} = \frac{1}{na^2 \sqrt{1-e^2}} \left(\frac{1-e^2}{e} \frac{\partial \mathcal{R}}{\partial e} - \frac{\cos i}{\sin i} \frac{\partial \mathcal{R}}{\partial i} \right)$$

$$\frac{dM}{dt} - n = \frac{1}{na^2} \left(-2a \frac{\partial \mathcal{R}}{\partial a} - \frac{1-e^2}{e} \frac{\partial \mathcal{R}}{\partial e} \right)$$

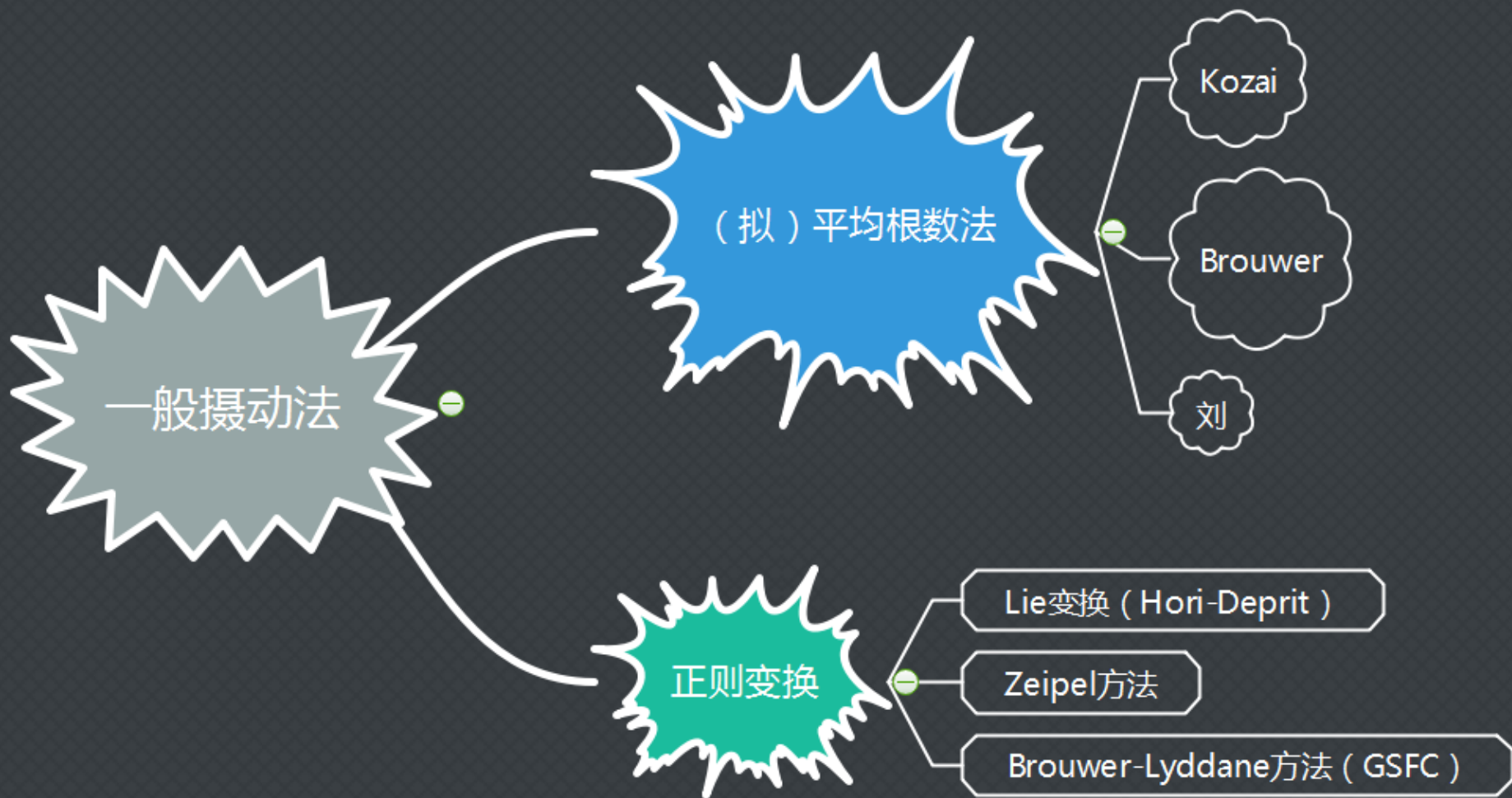
高斯型 (S、T、W)

$$\left\{ \begin{aligned} \frac{da}{dt} &= \frac{2}{n\sqrt{1-e^2}} [Se \sin f + T(1+e \cos f)] \\ \frac{de}{dt} &= \frac{\sqrt{1-e^2}}{na} [S \sin f + T(\cos f + \cos E)] \\ \frac{di}{dt} &= \frac{r \cos u}{na^2 \sqrt{1-e^2}} W \\ \frac{d\Omega}{dt} &= \frac{r \sin u}{na^2 \sqrt{1-e^2} \sin i} W \\ \frac{d\omega}{dt} &= \frac{\sqrt{1-e^2}}{nae} \left[-S \cos f + T \left(1 + \frac{r}{p} \right) \sin f \right] - \cos i \frac{d\Omega}{dt} \\ \frac{dM}{dt} &= n - \frac{1-e^2}{nae} \left[-S \left(\cos f - 2e \frac{r}{p} \right) + T \left(1 + \frac{r}{p} \right) \sin f \right] \end{aligned} \right.$$

小偏心率小倾角高斯型根数摄动方程

$$\left. \begin{aligned}
 \frac{da}{dt} &= \frac{2}{n\sqrt{1-e^2}} \left[S(\xi \sin u - \eta \cos u) + T\left(\frac{p}{r}\right) \right] \\
 \frac{d\xi}{dt} &= \frac{\sqrt{1-e^2}}{na} \left\{ S \sin u + T \left[(\cos \tilde{u} + \cos u) - \frac{\eta}{\sqrt{1-e^2}(1+\sqrt{1-e^2})} (\xi \sin \tilde{u} - \eta \cos \tilde{u}) \right] \right. \\
 &\quad \left. + W\left(\frac{r}{p}\right) \left[-\frac{\eta}{\cos(i/2)} (h \sin u - k \cos u) \right] \right\} \\
 \frac{d\eta}{dt} &= \frac{\sqrt{1-e^2}}{na} \left\{ -S \cos u + T \left[(\sin \tilde{u} + \sin u) + \frac{\xi}{\sqrt{1-e^2}(1+\sqrt{1-e^2})} (\xi \sin \tilde{u} - \eta \cos \tilde{u}) \right] \right. \\
 &\quad \left. + W\left(\frac{r}{p}\right) \left[\frac{\xi}{\cos(i/2)} (h \sin u - k \cos u) \right] \right\} \\
 \frac{dh}{dt} &= \frac{\sqrt{1-e^2}}{2na \cos(i/2)} W\left(\frac{r}{p}\right) [\cos u - h(h \cos u + k \sin u)] \\
 \frac{dk}{dt} &= \frac{\sqrt{1-e^2}}{2na \cos(i/2)} W\left(\frac{r}{p}\right) [\sin u - k(h \cos u + k \sin u)] \\
 \frac{d\lambda}{dt} &= n - \frac{\sqrt{1-e^2}}{na} \left\{ 2S\sqrt{1-e^2} \left(\frac{r}{p}\right) + \frac{1}{1+\sqrt{1-e^2}} \left[S(\xi \cos u + \eta \sin u) - T\left(1 + \frac{r}{p}\right) (\xi \sin u - \eta \cos u) \right] \right. \\
 &\quad \left. - W\left(\frac{r}{p}\right) \left(\frac{h \sin u - k \cos u}{\cos(i/2)} \right) \right\}
 \end{aligned} \right\}$$

一般摄动法



摄动法构造 (Liu)

$$\frac{d\sigma}{dt} = f_0(a) + f_1(\sigma, t, \varepsilon)$$

$$\left\{ \begin{array}{l} f_0(a) = \delta n \\ \delta = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} = (0 \ 0 \ 0 \ 0 \ 0 \ 1)^T \end{array} \right.$$

$$a^{(0)}(t) = a_0, \quad e^{(0)}(t) = e_0, \quad i^{(0)}(t) = i_0,$$

$$\Omega^{(0)}(t) = \Omega_0, \quad \omega^{(0)}(t) = \omega_0$$

$$M^{(0)}(t) = M_0 + n_0(t - t_0)$$

$$\sigma(t) = \sigma^{(0)}(t) + \Delta\sigma^{(1)}(t, \varepsilon) + \Delta\sigma^{(2)}(t, \varepsilon^2) + \cdots + \Delta\sigma^{(l)}(t, \varepsilon^l) + \cdots$$

$$\Delta\sigma^{(l)}(t, \varepsilon^l) = \varepsilon^l \beta_l(t), \quad l = 1, 2, \cdots$$

摄动法构造

$$\begin{aligned} & \frac{d}{dt} \left[\sigma^{(0)} + \Delta\sigma^{(1)} + \Delta\sigma^{(2)} + \dots + \Delta\sigma^{(l)} + \dots \right] \\ &= f_0(a) + \frac{\partial f_0}{\partial a} \left[\Delta a^{(1)} + \Delta a^{(2)} + \dots \right] + \frac{1}{2} \frac{\partial^2 f_0}{\partial a^2} \left[\Delta a^{(1)} + \dots \right]^2 + \dots \\ &+ f_1(\sigma, t, \varepsilon) + \sum_{j=1}^6 \frac{\partial f_1}{\partial \sigma_j} \left[\Delta\sigma_j^{(1)} + \Delta\sigma_j^{(2)} + \dots \right] \\ &+ \frac{1}{2} \sum_{j=1}^6 \sum_{k=1}^6 \frac{\partial^2 f_1}{\partial \sigma_j \partial \sigma_k} \left[\Delta\sigma_j^{(1)} + \dots \right] \left[\Delta\sigma_k^{(1)} + \dots \right] + \dots \end{aligned}$$

$$\begin{cases} \sigma^{(0)}(t) = \sigma_0 + \delta n_0(t - t_0) \\ \Delta\sigma^{(1)}(t) = \int_{t_0}^t \left[\delta \frac{\partial n}{\partial a} \Delta a^{(1)} + f_1(\sigma, t, \varepsilon)_1 \right]_{\sigma^{(0)}} dt \\ \Delta\sigma^{(2)}(t) = \int_{t_0}^t \left[\delta \left(\frac{\partial n}{\partial a} \Delta a^{(2)} + \frac{1}{2} \frac{\partial^2 n}{\partial a^2} (\Delta a^{(1)})^2 \right) + \sum_j \frac{\partial f_1}{\partial \sigma_j} \Delta\sigma_j^{(1)} \right]_{\sigma^{(0)}} dt \\ \dots \end{cases}$$

J2摄动计算范例

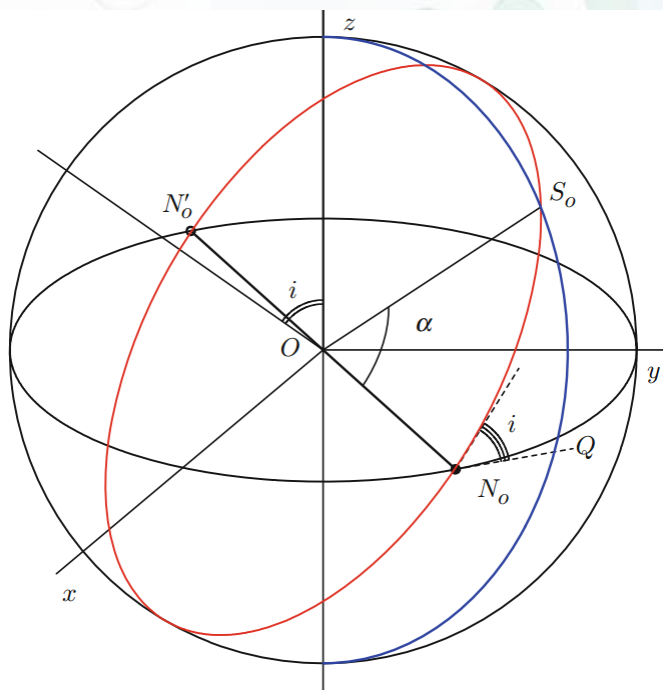
$$U = U_0 + \mathcal{R}$$

$$\mathcal{R} = -\frac{\mu R^2}{r^3} J_2 \frac{3 \sin^2 \psi - 1}{2}$$

$$\sin \psi = \sin i \sin(\omega + v)$$

$$\langle \mathcal{R} \rangle = \frac{1}{T} \int_0^T \mathcal{R} dt = \frac{1}{2\pi} \int_0^{2\pi} \mathcal{R} dM$$

$$\langle \mathcal{R}_2 \rangle = \langle \mathcal{R} \rangle = -\frac{1}{4} \frac{\mu R^2}{a^3 (1 - e^2)^{3/2}} J_2 (3 \sin^2 i - 2)$$



J2摄动范例（一阶长期项）

$$\mathcal{R}(v) = \mathcal{R}_s + \mathcal{R}_p(v)$$

$$\frac{\partial \mathcal{R}_s}{\partial a} = -\frac{3}{a} \mathcal{R}_s, \quad \frac{\partial \mathcal{R}_s}{\partial e} = \frac{3e}{1-e^2} \mathcal{R}_s, \quad \frac{\partial \mathcal{R}_s}{\partial i} = \frac{6 \sin i \cos i}{3 \sin^2 i - 2} \mathcal{R}_s$$

$$\dot{a} = 0,$$

$$\dot{e} = 0,$$

$$\dot{i} = 0,$$

$$\dot{\Omega} = -\frac{3}{2(1-e^2)^2} n J_2 \left(\frac{R}{a} \right)^2 \cos i$$

$$\dot{\omega} = \frac{3}{4(1-e^2)^2} n J_2 \left(\frac{R}{a} \right)^2 (5 \cos^2 i - 1),$$

$$\dot{M} - n = \Delta n = \frac{3}{4(1-e^2)^{3/2}} n J_2 \left(\frac{R}{a} \right)^2 (3 \cos^2 i - 1)$$

$$a(t) = a_0, \quad e(t) = e_0, \quad i(t) = i_0$$

$$\Omega(t) = \Omega_0 + \dot{\Omega} t, \quad \omega(t) = \omega_0 + \dot{\omega} t,$$

$$M(t) = M_0 + nt + (\Delta n)t.$$

级数方法摄动法步骤（林家翘）

假定因变量可以展为小参数的幂级数

- 第一步：把幂级数带入微分方程
- 第二步：把所有量展开，使得每一项都可以写为一个幂级数。
- 第三步：把方程中所有同幂次项收集起来，并让级数中各幂次的系数等于零。
- 第四步：把幂级数代入原先的边界或初始条件，展开系数等于零。这样就得到一组隶属于第三步所得的一系列微分方程的初值或边界条件。
- 第五步：相继求解第一步至第四步的一系列微分方程和边界条件。

常微分方程主要摄动法（林家翘）

- 级数方法
- 参数微商法
- 逐次逼近法
-

当函数满足

- ◆ 代数方程
- ◆ 差分方程
- ◆ 偏微分方程
- ◆ 积分方程

等情况时，摄动法思想与手段是类似的。

拟平均根数法 (Liu)

拟平均根数定义

$$\bar{\sigma} = \bar{\sigma}_0 + (\sigma_1 + \sigma_2)(t - t_0) + [\sigma_i^{(1)}(t) - \sigma_i^{(1)}(t_0)]$$

选择无奇点根数

$$\bar{\sigma}_0 = \sigma_0 - [\sigma_s^{(1)}(t_0) + \sigma_s^{(2)}(t_0)]$$

$$\sigma = (a, i, \Omega, \xi = e \cos \omega, \eta = -e \sin \omega, \lambda = M + \omega)$$

计算步骤

1) 由 $\bar{\sigma}_0$ 计算 $\sigma_1(t-t_0)$ ，并由下式定义的平根数给出计算中用到的 $\bar{\sigma}$ ：

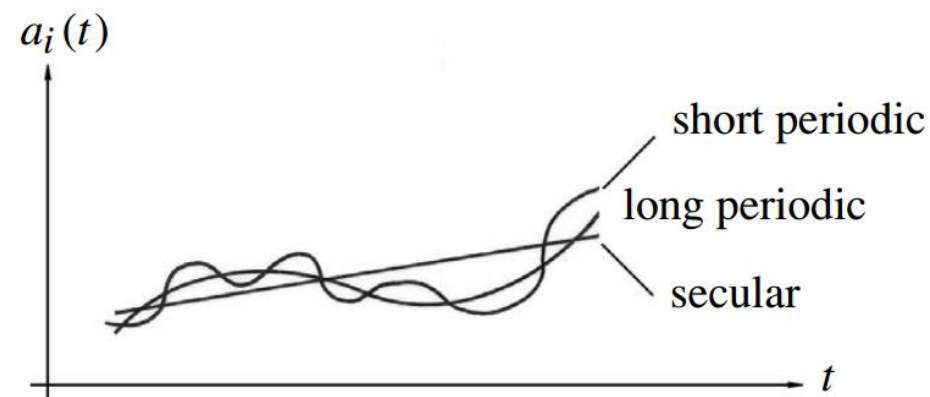
$$\left. \begin{aligned} \bar{a} &= \bar{a}_0 & \bar{i} &= \bar{i}_0 \\ \bar{\Omega} &= \bar{\Omega}_0 + \Omega_1(t-t_0) \\ \bar{\xi} &= \bar{\xi}_0 \cos \omega_1(t-t_0) + \bar{\eta}_0 \sin \omega_1(t-t_0) \\ \bar{\eta} &= \bar{\eta}_0 \cos \omega_1(t-t_0) - \bar{\xi}_0 \sin \omega_1(t-t_0) \\ \bar{\lambda} &= \bar{\lambda}_0 + (\bar{n}_0 + \lambda_1)(t-t_0) \end{aligned} \right\}$$

2) 由 $\bar{\sigma}_0$ 和 $\bar{\sigma}$ 计算 $\sigma_2(t-t_0)$ 和 $[\sigma_i^{(1)}(t) - \sigma_i^{(1)}(t_0)]$ ；

3) 由 $\bar{\sigma}$ 、 $\sigma_2(t-t_0)$ 和 $[\sigma_i^{(1)}(t) - \sigma_i^{(1)}(t_0)]$ 计算出真正的平根数 $\bar{\sigma}$ ，即：

$$\bar{\sigma} = \bar{\sigma}_0 + (\sigma_1 + \sigma_2)(t - t_0) + [\sigma_i^{(1)}(t) - \sigma_i^{(1)}(t_0)]$$

4) 由 $\bar{\sigma}$ 计算 $\sigma_s^{(1)}(t), \sigma_s^{(2)}(t)$ ，最后给出瞬根数 $\sigma = \bar{\sigma} + \sigma_s^{(1)}(t) + \sigma_s^{(2)}(t)$ 。



一阶长期项

$$a_1 = e_1 = i_1 = 0$$

$$\Omega_1 = -\frac{3}{2} \frac{J_2}{P^2} n \cos i$$

$$\omega_1 = \frac{3}{4} \frac{J_2}{P^2} n (4 - 5 \sin^2 i)$$

$$\lambda_1 = \frac{3}{4} \frac{J_2}{P^2} n \left[(2 - 3 \sin^2 i) \sqrt{1 - e^2} + (4 - 5 \sin^2 i) \right]$$

一阶短周期项

$$a_s^{(1)} = \frac{J_2}{a} \left\{ \left(1 - \frac{3}{2} \sin^2 i \right) \left[\left(\frac{a}{r} \right)^3 - (1 - e^2)^{-\frac{3}{2}} \right] + \frac{3}{2} \left(\frac{a}{r} \right)^3 \sin^2 i \cos 2u \right\}$$

$$i_s^{(1)} = \frac{J_2}{8P^2} \sin 2i \left\{ 3 \cos 2u + 3(\xi \cos u + \eta \sin u) \right. \\ \left. + (\xi \cos 3u - \eta \sin 3u) + (\xi^2 - \eta^2) F_2(e) \right\}$$

$$\Omega_s^{(1)} = \frac{3J_2}{2P^2} \cos i \left\{ -(u - \lambda) + \frac{1}{2} \sin 2u - \frac{1}{2} (\xi \sin u + 3\eta \cos u) \right. \\ \left. + \frac{1}{6} (\xi \sin 3u + \eta \cos 3u) - \frac{1}{3} \xi \eta F_2(e) \right\}$$

一阶短周期项

$$\begin{aligned}\xi_s^{(1)} = & \frac{3}{2} \frac{J_2}{P^2} \left\{ \left(2 - \frac{5}{2} \sin^2 i \right) (u - \lambda) \eta + \left(1 + \frac{1}{4} \xi^2 + \frac{9}{4} \eta^2 \right) \cos u \right. \\ & + \xi \eta \sin u + \frac{1}{2} \xi \cos 2u - \eta \sin 2u + \frac{1}{12} (\xi^2 - 3\eta^2) \cos 3u \\ & - \frac{1}{3} \xi \eta \sin 3u + \left(1 - \frac{e^2}{12} F_2(e) \right) \xi - \frac{1}{12} F_2(e) (\xi^2 - 3\eta^2) \xi \\ & + \sin^2 i \left[\left(-\frac{5}{4} + \frac{3}{8} \xi^2 - \frac{25}{8} \eta^2 \right) \cos u - \frac{1}{4} \xi \eta \sin u \right. \\ & + \frac{1}{2} \xi \cos 2u + 2\eta \sin 2u + \left(\frac{7}{12} + \frac{11}{48} \xi^2 + \frac{25}{48} \eta^2 \right) \cos 3u \\ & + \frac{7}{24} \xi \eta \sin 3u + \frac{3}{8} \xi \cos 4u - \frac{3}{8} \eta \sin 4u + \frac{1}{16} (\xi^2 - \eta^2) \cos 5u \\ & \left. \left. - \frac{1}{8} \xi \eta \sin 5u - \left(\frac{5}{4} - \frac{e^2}{6} F_2(e) \right) \xi + \left(\frac{F_2(e)}{4} - \frac{F_3(e)}{6} \right) (\xi^2 - 3\eta^2) \xi \right] \right\}\end{aligned}$$

一阶短周期项

$$\begin{aligned}\eta_s^{(1)} = & \frac{3}{2} \frac{J_2}{P^2} \left\{ - \left(2 - \frac{5}{2} \sin^2 i \right) (u - \lambda) \xi - \left(1 + \frac{5}{4} \xi^2 + \frac{1}{4} \eta^2 \right) \sin u \right. \\ & - 2 \xi \eta \cos u - \frac{1}{2} \eta \cos 2u + \frac{1}{12} e^2 \sin 3u + \\ & \left. \left(1 - \frac{e^2}{12} F_2(e) \right) \eta + \frac{1}{12} F_2(e) (\eta^2 - 3\xi^2) \eta \right. \\ & + \sin^2 i \left[\left(\frac{7}{4} + \frac{9}{8} \xi^2 + \frac{9}{8} \eta^2 \right) \sin u + \frac{13}{4} \xi \eta \cos u - \frac{1}{2} \xi \sin 2u \right. \\ & + 2 \eta \cos 2u - \left. \left(\frac{7}{12} + \frac{13}{48} \xi^2 + \frac{23}{48} \eta^2 \right) \sin 3u \right. \\ & + \frac{5}{24} \xi \eta \cos 3u - \frac{3}{8} \eta \cos 4u - \frac{3}{8} \xi \sin 4u - \frac{1}{16} (\xi^2 - \eta^2) \sin 5u \\ & \left. \left. - \frac{1}{8} \xi \eta \cos 5u - \left(\frac{5}{4} - \frac{e^2}{6} F_2(e) \right) \eta + \left(\frac{F_2(e)}{4} - \frac{F_3(e)}{6} \right) (3\xi^2 - \eta^2) \eta \right] \right\}.\end{aligned}$$

一阶短周期项

$$\begin{aligned}
 \lambda_s^{(1)} = & \frac{3}{2} \frac{J_2}{P^2} \left\{ \left(2 - \frac{5}{2} \sin^2 i \right) (u - \lambda) - \frac{1}{2} \sin 2u + \frac{3}{2} \xi \sin u + \frac{5}{2} \eta \cos u \right. \\
 & - \frac{1}{6} (\xi \sin 3u + \eta \cos 3u) + \frac{1}{1 + \sqrt{1 - e^2}} \left[\left(1 - \frac{1}{4} e^2 \right) (\xi \sin u + \eta \cos u) \right. \\
 & + \frac{1}{2} (\xi^2 - \eta^2) \sin 2u + \xi \eta \cos 2u + \frac{1}{12} (\xi^2 - 3\eta^2) \xi \sin 3u \\
 & \left. + \frac{1}{12} (3\xi^2 - \eta^2) \eta \cos 3u \right] + \frac{1}{3} \xi \eta F_2(e) \left. \right\} \\
 & + \frac{3}{2} \frac{J_2}{P^2} \sin^2 i \left\{ \frac{5}{4} \sin 2u - \frac{5}{2} (\xi \sin u + \eta \cos u) - \frac{5}{6} \xi \eta F_2(e) \right. \\
 & + \frac{1}{1 + \sqrt{1 - e^2}} \left[\left(-\frac{1}{2} + \frac{5}{4} \sqrt{1 - e^2} + \frac{1}{8} (\xi^2 - \eta^2) \right) \xi \sin u \right. \\
 & + \left(-\frac{5}{2} - \frac{5}{4} \sqrt{1 - e^2} + \frac{1}{8} (7\xi^2 + 5\eta^2) \right) \eta \cos u - \frac{3}{2} \xi \eta \cos 2u \\
 & - \frac{3}{4} (\xi^2 - \eta^2) \sin 2u + \left(1 + \frac{5}{12} \sqrt{1 - e^2} - \frac{7}{48} \xi^2 + \frac{17}{48} \eta^2 \right) \xi \sin 3u \\
 & \left. + \left(1 + \frac{5}{12} \sqrt{1 - e^2} - \frac{19}{48} \xi^2 + \frac{5}{48} \eta^2 \right) \eta \cos 3u + \frac{3}{8} (\xi^2 - \eta^2) \sin 4u \right. \\
 & + \frac{3}{4} \xi \eta \cos 4u + \frac{1}{16} (\xi^2 - 3\eta^2) \xi \sin 5u + \frac{1}{16} (3\xi^2 - \eta^2) \eta \cos 5u \\
 & \left. + \left(\frac{1}{4} + \left(\frac{1}{3} + \frac{e^2}{6} \right) F_2(e) \right) \xi \eta \right\}
 \end{aligned}$$

$$F_2(e) = \frac{\overline{\cos 2f}}{e^2}, \quad F_3(e) = \frac{\overline{\cos 2f} - 3e^2/4}{e^4}$$

一阶长周期项及二阶项略，参考文献：
《航天器轨道理论》，刘林，
国防工业出版社，2000

Draper Semianalytical Satellite Theory (DSST) 半解析算法

- ▷ AbstractGaussianContribution.java
- ▷ AbstractGaussianContributionContext.java
- ▷ DSSTAtmosphericDrag.java
- ▷ DSSTForceModel.java
- ▷ DSSTNewtonianAttraction.java
- ▷ DSSTNewtonianAttractionContext.java
- ▷ DSSTSolarRadiationPressure.java
- ▷ DSSTTesseral.java
- ▷ DSSTTesseralContext.java
- ▷ DSSTThirdBody.java
- ▷ DSSTThirdBodyContext.java
- ▷ DSSTZonal.java
- ▷ DSSTZonalContext.java
- ▷ FieldAbstractGaussianContributionContext.java
- ▷ FieldDSSTNewtonianAttractionContext.java
- ▷ FieldDSSTTesseralContext.java
- ▷ FieldDSSTThirdBodyContext.java
- ▷ FieldDSSTZonalContext.java
- ▷ FieldForceModelContext.java
- ▷ FieldShortPeriodTerms.java
- ▷ ForceModelContext.java
- ▷ package-info.java
- ▷ ShortPeriodTerms.java

动力学模型

- ▷ DSSTDSConverter.java
- ▷ DSSTJacobiansMapper.java
 - ▷ DSSTJacobiansMapper
 - ▷ I
 - ▷ STATE_DIMENSION
 - ▷ map
 - ▷ name
 - ▷ parameters
 - ▷ propagationType
 - ▷ propagator
 - ▷ shortPeriodDerivatives
 - ▷ DSSTJacobiansMapper(String, ParameterDrivers)
 - ▷ addToRow(double[], int, double[][]): void
 - ▷ getAdditionalStateDimension(): int
 - ▷ getConversionJacobian(SpacecraftState): double[][]
 - ▷ getParametersJacobian(SpacecraftState, double): double[][]
 - ▷ getStateJacobian(SpacecraftState, double[][]): double[][]
 - ▷ setInitialJacobians(SpacecraftState, double[][]): void
 - ▷ setShortPeriodJacobians(SpacecraftState): void
 - ▷ DSSTPartialDerivativesEquations.java
 - ▷ DSSTPropagator.java
 - ▷ FieldDSSTPropagator.java

偏导数部分

半分析带谐项解

$$\dot{a}_{sec} = \dot{e}_{sec} = di/dt_{sec} = 0$$

$$\dot{\Omega}_{sec} = n \cos i \sum_{\substack{n \geq 2 \\ \text{step 2}}} -J_n \left(\frac{r_e}{p} \right)^n K_{n1}(e) \sum_{q=1}^{n/2} (-1)^{\frac{1}{2}(n+2q)} \frac{2q}{2^{n+2q}} \binom{n}{\frac{n}{2}-q} \binom{n+2q}{n} \binom{2q}{q} (\sin i)^{2q-2}$$

$$\dot{\omega}_{sec} = [\dot{\omega}]_c - \cos i \cdot \dot{\Omega}_{sec}$$

$$[\dot{\omega}]_c = n \sum_{\substack{n \geq 2 \\ \text{step 2}}} -J_n \left(\frac{r_e}{p} \right)^n \left[\sum_{q=0}^{n/2} (-1)^{\frac{1}{2}(n+2q)} 2^{-(n+2q)} \binom{n}{\frac{n}{2}-q} \binom{n+2q}{n} \binom{2q}{q} (\sin i)^{2q} \right] \\ \cdot \left[(1 - e^2) \left(\frac{l}{e^2} K_{n1}(e) \right) + (2n - 1) K_{n1}(e) \right]$$

$$\dot{M}_{sec} = n \sqrt{1 - e^2} \sum_{\substack{n \geq 2 \\ \text{step 2}}} -J_n \left(\frac{r_e}{p} \right)^n K_{n1}(e)$$

$$\cdot \sum_{q=0}^{n/2} (-1)^{\frac{1}{2}(n+2q)} \frac{2(n+1)}{2^{n+2q}} \binom{n}{\frac{n}{2}-q} \binom{n+2q}{n} \binom{2q}{q} (\sin i)^{2q} - \sqrt{1 - e^2} [\dot{\omega}]_c$$

$$K_{n1}(e) = \sum_{\substack{l=0 \\ \text{step 2}}}^{n-2} \binom{n-1}{l} \binom{l}{l/2} \left(\frac{e}{2} \right)^l$$

$$\frac{l}{e^2} K_{n1}(e) = \sum_{\substack{l=0 \\ \text{step 2}}}^{n-2} \frac{l}{4} \binom{n-1}{l} \binom{l}{l/2} \left(\frac{e}{2} \right)^{l-2}$$

半分析长期项

$$\dot{a}_{long} = 0$$

$$\dot{e}_{long} = - \left(\frac{1 - e^2}{e} \tan i \right) di/dt_{long}$$

$$di/dt_{long} = n \cos i \sum_{n \geq 3} -J_n \left(\frac{r_e}{p} \right)^n \sum_{p=0}^{(n-2+\delta_1)/2} K_{n1}^p(e) (n-2p)$$

$$\times [-(1-\delta_1) \sin(n-2p)\omega + \delta_1 \cos(n-2p)\omega]$$

$$\times \left[\sum_{q=0}^p (-1)^{\frac{1}{2}(n+2q-\delta_1)} 2^{-(2n-2p+2q-1)} \binom{n}{p-q} \binom{2n-2p+2q}{n} \binom{n-2p+2q}{q} (\sin i)^{n-2p+2q-1} \right]$$

$$\dot{\Omega}_{long} = n \cos i \sum_{n \geq 3} -J_n \left(\frac{r_e}{p} \right)^n \sum_{p=0}^{(n-2+\delta_1)/2} K_{n1}^p(e) [(1-\delta_1) \cos(n-2p)\omega + \delta_1 \sin(n-2p)\omega]$$

$$\times \left[\sum_{q=0}^p (-1)^{\frac{1}{2}(n+2q-\delta_1)} \frac{(n-2p+2q)}{2^{(2n-2p+2q-1)}} \binom{n}{p-q} \binom{2n-2p+2q}{n} \binom{n-2p+2q}{q} (\sin i)^{n-2p+2q-2} \right]$$

$$\dot{\omega}_{long} = [\dot{\omega}]_l - \cos i \times \dot{\Omega}_{long}$$

半分析长期项

$$[\dot{\omega}]_l = n \sum_{n \geq 3} -J_n \left(\frac{r_e}{p} \right)^n$$

$$\times \sum_{p=0}^{(n-2+\delta_1)/2} \left[(1-e^2) \left(\frac{l}{e^2} K_{n1}^p(e) \right) + (2n-1) K_{n1}^p(e) \right]$$

$$\times [(1-\delta_1) \cos(n-2p)\omega + \delta_1 \sin(n-2p)\omega]$$

$$\times \left[\sum_{q=0}^p (-1)^{\frac{1}{2}(n+2q-\delta_1)} 2^{-(2n-2p+2q-1)} \binom{n}{p-q} \binom{2n-2p+2q}{n} \binom{n-2p+2q}{q} (\sin i)^{n-2p+2q} \right]$$

$$\dot{M}_{long} = -\sqrt{1-e^2} [\dot{\omega}]_l + n \sqrt{1-e^2} \sum_{n \geq 3} -J_n \left(\frac{r_e}{p} \right)^n$$

$$\times \sum_{p=0}^{(n-2+\delta_1)/2} K_{n1}^p(e) [(1-\delta_1) \cos(n-2p)\omega + \delta_1 \sin(n-2p)\omega]$$

$$\times \left[\sum_{q=0}^p (-1)^{\frac{1}{2}(n+2q-\delta_1)} \frac{(n+1)}{2^{(2n-2p+2q-2)}} \binom{n}{p-q} \binom{2n-2p+2q}{n} \binom{n-2p+2q}{q} (\sin i)^{n-2p+2q} \right]$$

$$\delta_1 = \frac{1}{2} [1 - (-1)^n], \delta_2 = \begin{cases} 0 & p = 0 \\ 1 & p \neq 0 \end{cases}$$

$$K_{n1}^p(e) = \delta_2 \sum_{\substack{l=n-2p \\ \text{step 2}}}^{n-2} \binom{n-1}{l} \binom{l}{\frac{1}{2}(l-n+2p)} \left(\frac{e}{2} \right)^l$$

$$\frac{l}{e^2} K_{n1}^p(e) = \delta_2 \sum_{\substack{l=n-2p \\ \text{step 2}}}^{n-2} \frac{l}{4} \binom{n-1}{l} \binom{l}{\frac{1}{2}(l-n+2p)} \left(\frac{e}{2} \right)^{l-2}$$

二阶影响

$$\dot{a}_{long,22} = \dot{e}_{long,22} = \dot{a}_{long,31} = \dot{a}_{long,32} = \dot{a}_{long,33} = 0$$

$$di/dt_{long,22} = 3 \left(\frac{r_e}{p} \right)^2 n \sin i (C_{22} \sin 2\Omega_s - S_{22} \cos 2\Omega_s)$$

$$\dot{\Omega}_{long,22} = 3 \left(\frac{r_e}{p} \right)^2 n \cos i (C_{22} \cos 2\Omega_s + S_{22} \sin 2\Omega_s)$$

$$\dot{\omega}_{long,22} = \frac{3}{2} \left(\frac{r_e}{p} \right)^2 n (5\sin^2 i - 2) (C_{22} \cos 2\Omega_s + S_{22} \sin 2\Omega_s)$$

$$\dot{M}_{long,22} = \frac{9}{2} \left(\frac{r_e}{p} \right)^2 n \sin^2 i (1 - e^2)^{\frac{1}{2}} (C_{22} \cos 2\Omega_s + S_{22} \sin 2\Omega_s)$$

$$\dot{e}_{long,31} = \frac{3}{8} \left(\frac{r_e}{p} \right)^3 n (1 - e^2)$$

$$\times \{ C_{31} [\sin \omega \cos \Omega_s (5\sin^2 i - 4) + \cos i \cos \omega \sin \Omega_s (15\sin^2 i - 4)] \\ + S_{31} [\sin \omega \sin \Omega_s (5\sin^2 i - 4) + \cos i \cos \omega \cos \Omega_s (4 - 15\sin^2 i)] \}$$

$$di/dt_{long,31} = \frac{3}{8} \left(\frac{r_e}{p} \right)^3 n e \sin i \{ C_{31} [10 \cos i \sin \omega \cos \Omega_s + \cos \omega \sin \Omega_s (1 - 15\cos^2 i)] \\ + S_{31} [10 \cos i \sin \omega \sin \Omega_s + \cos \omega \cos \Omega_s (15\cos^2 i - 1)] \}$$

二阶影响

$$\dot{\Omega}_{long,31} = \frac{3}{8} \left(\frac{r_e}{p} \right)^3 ne \{ C_{31} [10 \cos i \cos \omega \cos \Omega_s + \sin \omega \sin \Omega_s (11 - 45 \cos^2 i)] \\ + S_{31} [10 \cos i \cos \omega \sin \Omega_s + \sin \omega \cos \Omega_s (45 \cos^2 i - 11)] \}$$

$$\dot{\omega}_{long,31} = -\frac{3}{8} \left(\frac{r_e}{p} \right)^3 n \left(4e + \frac{1}{e} \right) \\ \times \{ C_{31} [\cos \omega \cos \Omega_s (4 - 5 \sin^2 i) + \cos i \sin \omega \sin \Omega_s (15 \sin^2 i - 4)] \\ + S_{31} [\cos \omega \sin \Omega_s (4 - 5 \sin^2 i) + \cos i \sin \omega \cos \Omega_s (4 - 15 \sin^2 i)] \} \\ - \dot{\Omega}_{31} \cos i$$

$$\dot{M}_{long,31} = \frac{3}{8} \left(\frac{r_e}{p} \right)^3 n (1 - e^2)^{\frac{1}{2}} \\ \times \{ C_{31} [\cos \Omega_s \cos \omega (5 \sin^2 i - 4) + \sin \Omega_s \sin \omega \cos i (4 - 15 \sin^2 i)] \\ + S_{31} [\sin \Omega_s \cos \omega (5 \sin^2 i - 1) + \cos \Omega_s \sin \omega \cos i (15 \sin^2 i - 4)] \} \\ \times \left(4e - \frac{1}{e} \right)$$

二阶影响

$$\dot{e}_{long,32} = -\frac{15}{4} \left(\frac{r_e}{p}\right)^3 n (1 - e^2) \sin^2 i$$

$$\times \{C_{32} [\cos \omega \cos 2\Omega_s (3\sin^2 i - 2) + 2 \cos i \sin \omega \sin 2\Omega_s] \\ + S_{32} [\cos \omega \sin 2\Omega_s (3\sin^2 i - 2) - 2 \cos i \sin \omega \cos \Omega_s]\}$$

$$di/dt_{long,32} = -\frac{15}{4} \left(\frac{r_e}{p}\right)^3 ne$$

$$\times \{C_{32} [\sin \omega \sin 2\Omega_s (2 - 4\sin^2 i) - \cos i \cos \omega \cos 2\Omega_s (2 + 3\sin^2 i)] \\ + S_{32} [\sin \omega \cos 2\Omega_s (4\sin^2 i - 2) - \cos i \cos \omega \sin 2\Omega_s (2 + 3\sin^2 i)]\}$$

$$\dot{e}_{long,33} = \frac{45}{4} \left(\frac{r_e}{p}\right)^3 n (1 - e^2) \{C_{33} [2 \cos 3\Omega_s \sin \omega \cos i - \sin 3\Omega_s \cos \omega (3\sin^2 i - 2)] \\ + S_{33} [2 \sin 3\Omega_s \sin \omega \cos i - \cos 3\Omega_s \cos \omega (2 - 3\sin^2 i)]\}$$

$$di/dt_{long,33} = \frac{45}{4} \left(\frac{r_e}{p}\right)^3 n \frac{e}{\sin i}$$

$$\times \{C_{33} [\cos 3\Omega_s \sin \omega (4 - 7\sin^2 i) + \sin 3\Omega_s \cos \omega \cos i (4 + 3\sin^2 i)] \\ + S_{33} [\sin 3\Omega_s \sin \omega (4 - 7\sin^2 i) - \cos 3\Omega_s \cos \omega \cos i (4 + 3\sin^2 i)]\}$$

$$\dot{\Omega}_{long,33} = \frac{45}{2} \left(\frac{r_e}{p}\right)^3 ne [C_{33} (3 \sin 3\Omega_s \sin \omega \cos i - \cos 3\Omega_s \cos \omega)$$

$$- S_{33} (3 \cos 3\Omega_s \sin \omega \cos i - \sin 3\Omega_s \cos \omega)]$$

二阶影响

$$\dot{\omega}_{long,33} = \frac{45}{4} \left(\frac{r_e}{p} \right)^3 n \left(4e + \frac{1}{e} \right) \{ C_{33} [2 \cos 3\Omega_s \cos \omega \cos i + \sin 3\Omega_s \sin \omega (3\sin^2 i - 2)] \\ + S_{33} [2 \sin 3\Omega_s \cos \omega \cos i + \cos 3\Omega_s \sin \omega (2 - 3\sin^2 i)] \} - \dot{\Omega}_{33} \cos i$$

$$\dot{M}_{long,33} = \frac{45}{4} \left(\frac{r_e}{p} \right)^3 n (1 - e^2)^{\frac{1}{2}} \left(4e - \frac{1}{e} \right) \\ \times \{ C_{33} [2 \cos 3\Omega_s \cos \omega \cos i + \sin 3\Omega_s \sin \omega (3\sin^2 i - 2)] \\ + S_{33} [2 \sin 3\Omega_s \cos \omega \cos i + \cos 3\Omega_s \sin \omega (2 - 3\sin^2 i)] \}$$

$$\tan m\lambda_{nm} = \frac{S_{nm}}{C_{nm}}$$

$$J_{nm} = -(C_{nm}^2 + S_{nm}^2)^{\frac{1}{2}}$$

扇谐与田谐

$$\begin{aligned} a_{short,22} = & -\frac{3}{2} \frac{J_{22}}{a} r_e^2 \\ & \times \left\{ (1 + \cos i)^2 \left[\frac{1}{1-\tau} \cos (2M + 2\omega + 2\Omega_e) - \frac{1}{2} e \left(\frac{1}{1-2\tau} \cos (M + 2\omega + 2\Omega_e) \right. \right. \right. \\ & \left. \left. \left. - \frac{21}{3-2\tau} \cos (3M + 2\omega + 2\Omega_e) \right) \right] + (1 - \cos i)^2 \left[\frac{1}{1+\tau} \cos (2m + 2\omega - 2\Omega_e) \right. \right. \\ & \left. \left. - \frac{e}{2} \left(\frac{1}{1+2\tau} \cos (M + 2\omega - 2\Omega_e) - \frac{21}{3-2\tau} \cos (3M + 2\omega - 2\Omega_e) \right) \right] \right. \\ & \left. + 3e \sin^2 i \left[\frac{1}{1-2\tau} \cos (M + 2\Omega_e) + \frac{1}{1+2\tau} \cos (M - 2\Omega_e) \right] \right\} \end{aligned}$$

扇谐与田谐

$$\begin{aligned} e_{short,22} = & -\frac{3}{4}J_{22}\left(\frac{r_e}{a}\right)^2 \\ & \times \left\{ (1 + \cos i)^2 \left[\frac{1}{2 - 4\tau} \cos(M + 2\omega + 2\Omega_e) + \frac{7}{6 - 4\tau} \cos(3M + 2\omega + 2\Omega_e) \right. \right. \\ & \left. \left. + \frac{e}{2} \left(\frac{1}{\tau - 1} \cos(2M + 2\omega + 2\Omega_e) + \frac{17}{2 - \tau} \cos(4M + 2\omega + 2\Omega_e) \right) \right] \right. \\ & + (1 - \cos i)^2 \left[\frac{1}{2 + 4\tau} (\cos(M + 2\omega - 2\Omega_e)) + \frac{7}{6 + 4\tau} \cos(3M + 2\omega - 2\Omega_e) \right. \\ & \left. \left. + \frac{e}{2} \left(-\frac{1}{1 + \tau} \cos(2M + 2\omega - 2\Omega_e) + \frac{17}{2 + \tau} \cos(4M + 2\omega - 2\Omega_e) \right) \right] \right. \\ & + 2\sin^2 i \left[\frac{3}{2 - 4\tau} \cos(M + 2\Omega_e) + \frac{3}{2 + 4\tau} \cos(M - 2\Omega_e) \right. \\ & \left. \left. + \frac{9e}{4} \left(\frac{1}{1 - \tau} \cos(2M + 2\Omega_e) + \frac{1}{1 + \tau} \cos(2M - 2\Omega_e) \right) \right] \right\} \end{aligned}$$

扇谐与田谐

$$i_{short,22} = -\frac{3}{4}J_{22}\left(\frac{r_e}{a}\right)^2 \sin i$$
$$\times \left\{ (1 + \cos i) \left[\frac{e}{1-2\tau} \cos(M + 2\omega + 2\Omega_e) - \frac{7e}{3-2\tau} \cos(3M + 2\omega - 2\Omega_e) \right. \right.$$
$$\left. - \frac{1}{1-\tau} \cos(2M + 2\omega + 2\Omega_e) \right] - (1 - \cos i) \left[\frac{e}{1+2\tau} \cos(M + 2\omega - 2\Omega_e) \right.$$
$$\left. - \frac{7e}{3+2\tau} \cos(3M + 2\omega - 2\Omega_e) - \frac{1}{1+\tau} \cos(2M + 2\omega - 2\Omega_e) \right]$$
$$\left. - 6e \left[\frac{1}{1-2\tau} \cos(M + 2\Omega_e) - \frac{1}{1+2\tau} \cos(M - 2\Omega_e) \right] \right\}$$

$$\Omega_{short,22} = -\frac{3}{4}J_{22}\left(\frac{r_e}{a}\right)^2$$
$$\times \left\{ (1 + \cos i) \left[\frac{e}{1-2\tau} \sin(M + 2\omega + 2\Omega_e) - \frac{7e}{3-2\tau} \sin(3M + 2\omega + 2\Omega_e) \right. \right.$$
$$\left. - \frac{1}{1-\tau} \sin(2M + 2\omega + 2\Omega_e) \right] - (1 - \cos i) \left[\frac{e}{1+2\tau} \sin(M + 2\omega - 2\Omega_e) \right.$$
$$\left. - \frac{7e}{3+2\tau} \sin(3M + 2\omega - 2\Omega_e) - \frac{1}{1+\tau} \sin(2M + 2\omega - 2\Omega_e) \right]$$
$$\left. + 6e \cos i \left[\frac{1}{1-2\tau} \sin(M + 2\Omega_e) + \frac{1}{1+2\tau} \sin(M - 2\Omega_e) \right] \right\}$$

扇谐与田谐

$$\begin{aligned}\omega_{short,22} = & -\frac{3}{4} \frac{J_{22}}{e} \left(\frac{r_e}{a}\right)^2 \\ & \times \left\{ -(1 + \cos i)^2 \left[\frac{1}{2 - 4\tau} \sin(M + 2\omega + 2\Omega_e) - \frac{7}{6 - 4\tau} \sin(3M + 2\omega + 2\Omega_e) \right. \right. \\ & \left. \left. + \frac{5e}{2 - 2\tau} \sin(2M + 2\omega + 2\Omega_e) - \frac{17e}{4 - 2\tau} \sin(4M + 2\omega + 2\Omega_e) \right] \right. \\ & - (1 - \cos i)^2 \left[\frac{1}{2 + 4\tau} \sin(M + 2\omega - 2\Omega_e) - \frac{7}{6 + 4\tau} \sin(3M + 2\omega - 2\Omega_e) \right. \\ & \left. \left. + \frac{5e}{2 + 2\tau} \sin(2M + 2\omega - 2\Omega_e) - \frac{17e}{4 + 2\tau} \sin(4M + 2\omega - 2\Omega_e) \right] \right. \\ & + \frac{9}{2} e \sin^2 i \left[\frac{1}{1 - \tau} \sin(2M + 2\Omega_e) + \frac{1}{1 + \tau} \sin(2M - 2\Omega_e) \right] \\ & \left. \left. + 2 \sin^2 i \left[\frac{3}{2 - 4\tau} \sin(M + 2\Omega_e) + \frac{3}{2 + 4\tau} \sin(M - 2\Omega_e) \right] \right\} \\ & - \Omega_{short,22} \cos i\end{aligned}$$

扇谐与田谐

$$\begin{aligned} M_{short,22} = & -\frac{9}{4}J_{22}\left(\frac{r_e}{a}\right)^2 \\ & \times \left\{ (1 + \cos i)^2 \left[\frac{1}{1-\tau} \left(1 - \frac{1}{2-2\tau}\right) \sin(2M + 2\omega + 2\Omega_e) \right. \right. \\ & - \frac{e}{1-2\tau} \left(1 - \frac{1}{2-4\tau}\right) \sin(M + 2\omega + 2\Omega_e) \\ & \left. \left. - \frac{7e}{3-2\tau} \left(1 - \frac{3}{6-4\tau}\right) \sin(3M + 2\omega + 2\Omega_e) \right] \right. \\ & - (1 - \cos i)^2 \left[\frac{e}{1+2\tau} \left(1 - \frac{1}{2+4\tau}\right) \sin(M + 2\omega - 2\Omega_e) \right. \\ & - \frac{7e}{3+2\tau} \left(1 - \frac{3}{6+4\tau}\right) \sin(3M + 2\omega - 2\Omega_e) \\ & \left. \left. - \frac{1}{1+\tau} \left(1 - \frac{1}{2+2\tau}\right) \sin(2M + 2\omega - 2\Omega_e) \right] \right. \\ & + 6\sin^2 i \left[\frac{1}{1-2\tau} \left(1 - \frac{1}{2-4\tau}\right) \sin(M + 2\Omega_e) \right. \\ & \left. \left. + \frac{1}{1+2\tau} \left(1 - \frac{1}{1+2\tau}\right) \sin(M - 2\Omega_e) \right] \right\} \\ & - \omega_{short,22} - \Omega_{short,22} \cos i \end{aligned}$$

DSST半解析轨道计算软件 部分（主要）配置参数

```
# body.frame = CIO/2003-based ITRF accurate EOP  
body.frame = CIO/2010-based ITRF simple EOP  
# body.frame = CIO/2010-based ITRF accurate EOP
```

```
## date of the orbital parameters (UTC)  
orbit.date = 2011-12-12T11:57:20.000
```

```
### Keplerian elements  
## Semi-major axis (km)  
orbit.keplerian.a = 7204.535848109436  
## Eccentricity  
orbit.keplerian.e = 0.0012402238462686  
## Inclination (degrees)  
orbit.keplerian.i = 98.74341600466740  
## Right Ascension of Ascending Node (degrees)  
orbit.keplerian.raan = 43.32990110790340  
## Perigee Argument (degrees)  
orbit.keplerian.pa = 111.1990175076630  
## Anomaly (degrees)  
orbit.keplerian.aomaly = 68.66877509795670
```

```
# the degree value (7.292115e-5 rad/s) corresponds to week 1.0  
central.body.rotation.rate = 7.292115e-5  
## Central body gravity potential degree  
central.body.degree = 6  
## Central body gravity potential order  
central.body.order = 6  
## short period limits  
max.degree.zonal.short.periods = 6  
  
# typically min(max.degree.zonal.short.periods - 1, 4)  
max.eccentricity.power.zonal.short.periods = 4  
  
# typically 2 * max.degree.zonal.short.periods + 1  
max.frequency.true.longitude.zonal.short.periods = 13  
  
max.degree.tesseral.short.periods = 6  
max.order.tesseral.short.periods = 6  
  
# typically min(max.degree.tesseral.short.periods - 1, 4)  
max.eccentricity.power.tesseral.short.periods = 4  
  
# typically min(max.degree.tesseral.short.periods + max.eccentricity.power.tesseral.short.periods, 10)  
max.frequency.mean.longitude.tesseral.short.periods = 10  
  
max.degree.tesseral.m.dailies.short.periods = 6  
max.order.tesseral.m.dailies.short.periods = 6  
  
# typically min(max.degree.tesseral.short.periods - 2, 4)  
max.eccentricity.power.tesseral.m.dailies.short.periods = 4
```

DSST半解析轨道计算软件 部分（主要）配置参数

```
## duration (days)
duration.in.days = 365.0
## Time step between printed elements (seconds)
output.step = 86400.0

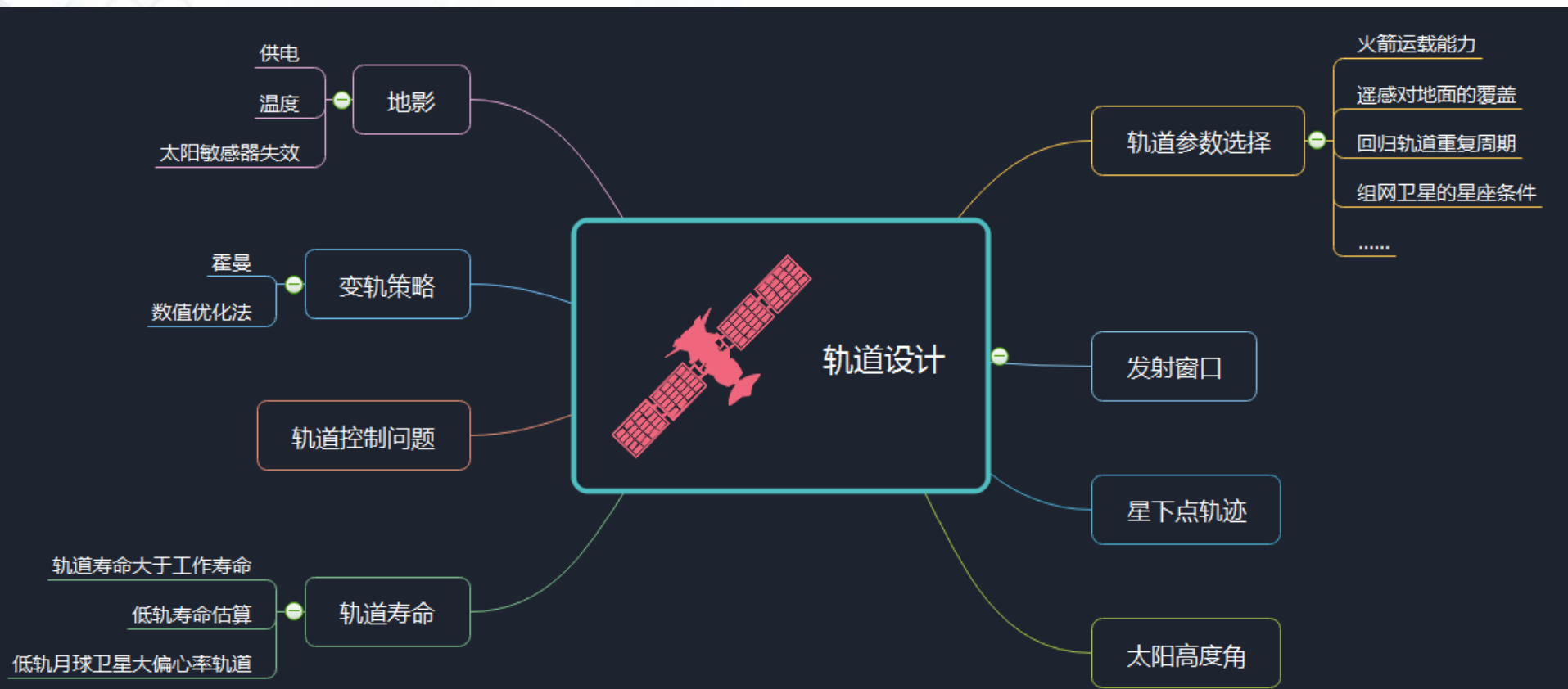
### Output parameters
## Output Keplerian parameters in output file
output.Keplerian = true
## Output equinoctial parameters in output file
output.equinoctial = true
## Output Cartesian parameters in output file
output.Cartesian = true
## Short period coefficients to output (commented out)
## in order to get the list of available coefficients
## run with a value set to all and then look at the output
## (a), (h), (k), (p), (q) and (L) parts do not
## describe the 6 columns that correspond to the short period
## Beware that the list is huge ...
output.short.period.coefficients = none
```

```
## seconds, and a position tolerance in meters (beware, 1 meter
## really the global error, it correspond to a local estimation
## at each step, often difficult to relate to final global error)
min.variable.integration.step = 6000.0
max.variable.integration.step = 86400.0
position.tolerance.variable.integration.step = 1.0

## interpolation grid specification
## only one of the two following options can be set: either
## of points or maximum gap. If both options are specified,
## will be generated.
## if neither option is set, a fixed number of 3 points will
# fixed number of interpolation points at each mean element
# fixed.number.of.interpolation.points = 3
# maximum gap between interpolation points
max.time.gap.between.interpolation.points = 86400.0
```

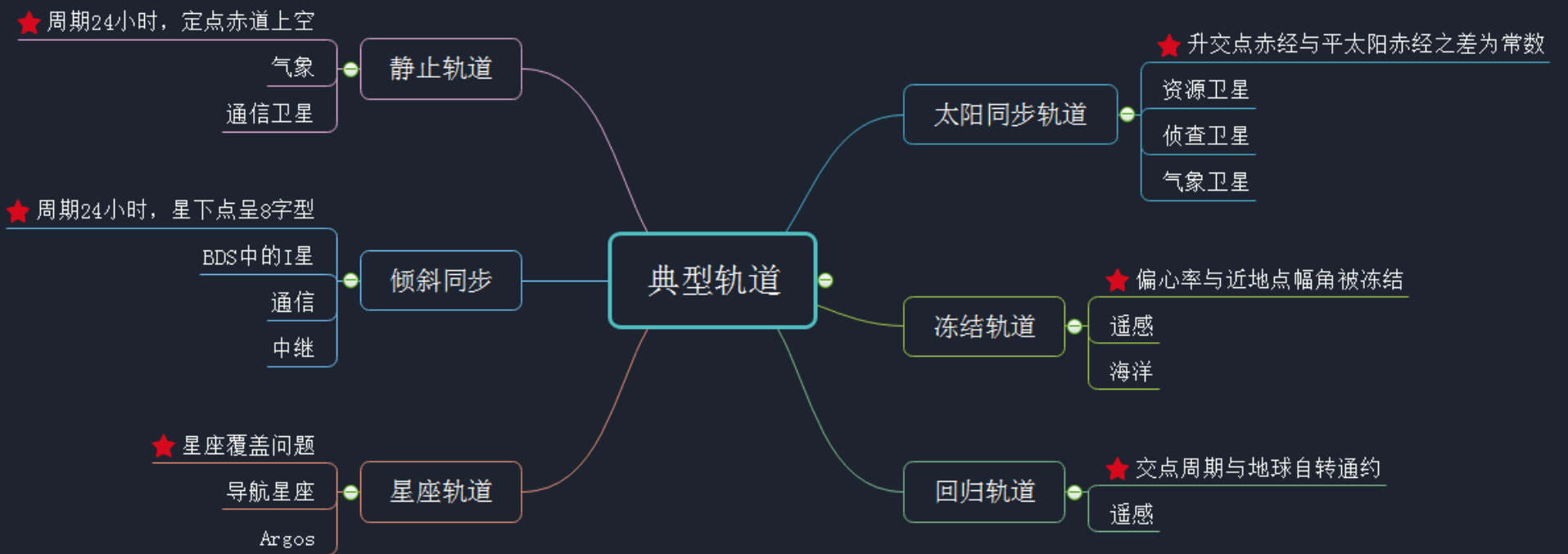
摄动法应用及轨道设计问题

轨道设计和卫星使命密切相关，不同用途的卫星，很难统一归纳轨道设计内容。下图是一些常见的地球卫星轨道设计共性问题。



轨道设计步骤与常用地球卫星轨道

- 问题提出
- 模型建立
- 数学求解
- 结果分析



作业问题：

1. 少部分同学未按时交作业
2. 作业无文档，只交了代码
3. 不需要把工程打包，只要代码本身。
4. 作业交PDF的，建议WORD。
5. 作业未做出来的，给了参考例子和程序，请补充完善。

