



中国科学院上海天文台

Shanghai Astronomical Observatory, Chinese Academy of Science



中国科学院大学

University of Chinese Academy of Sciences

轨道摄动力（一）

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主要内容

- 地球非球形摄动加速度
- 固体潮
- 海潮
- 大气潮
- 自转引起的附加摄动
- 第三体引力摄动

卫星在地固坐标系下引力加速度

$$V(r, \varphi, \lambda) = \frac{GM_E}{r} \left[1 + \sum_{l=2}^{\infty} \sum_{m=0}^l \left(\frac{a_E}{r} \right)^l \bar{P}_{lm}(\sin \varphi) [\bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda] \right]$$

$$\vec{f}_{CTS} = \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = \begin{pmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial y} \\ \frac{\partial V}{\partial z} \end{pmatrix} = \begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial \varphi}{\partial x} & \frac{\partial \lambda}{\partial x} \\ \frac{\partial r}{\partial y} & \frac{\partial \varphi}{\partial y} & \frac{\partial \lambda}{\partial y} \\ \frac{\partial r}{\partial z} & \frac{\partial \varphi}{\partial z} & \frac{\partial \lambda}{\partial z} \end{pmatrix} \begin{pmatrix} \frac{\partial V}{\partial r} \\ \frac{\partial V}{\partial \varphi} \\ \frac{\partial V}{\partial \lambda} \end{pmatrix} = \frac{\partial(r, \varphi, \lambda)^T}{\partial(x, y, z)} \cdot \begin{pmatrix} \frac{\partial V}{\partial r} \\ \frac{\partial V}{\partial \varphi} \\ \frac{\partial V}{\partial \lambda} \end{pmatrix}$$

引力位关于球坐标偏导数

$$\begin{cases} \frac{\partial V}{\partial r} = -\frac{GM_E}{r^2} \left[1 + \sum_{l=2}^{\infty} \sum_{m=0}^l (l+1) \left(\frac{a_E}{r} \right)^l \bar{P}_{lm}(\sin \varphi) [\bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda] \right] \\ \frac{\partial V}{\partial \varphi} = \frac{GM_E}{r} \left[\sum_{l=2}^{\infty} \sum_{m=0}^l \left(\frac{a_E}{r} \right)^l \beta \cdot \bar{P}_{l(m+1)}(\sin \varphi) - m \cdot \tan \varphi \cdot \bar{P}_{lm}(\sin \varphi) \cdot [\bar{C}_{lm} \cos m\lambda + \bar{S}_{lm} \sin m\lambda] \right] \\ \frac{\partial V}{\partial \lambda} = \frac{GM_E}{r} \left[\sum_{l=2}^{\infty} \sum_{m=0}^l m \cdot \left(\frac{a_E}{r} \right)^l \bar{P}_{lm}(\sin \varphi) [-\bar{C}_{lm} \sin m\lambda + \bar{S}_{lm} \cos m\lambda] \right] \end{cases}$$

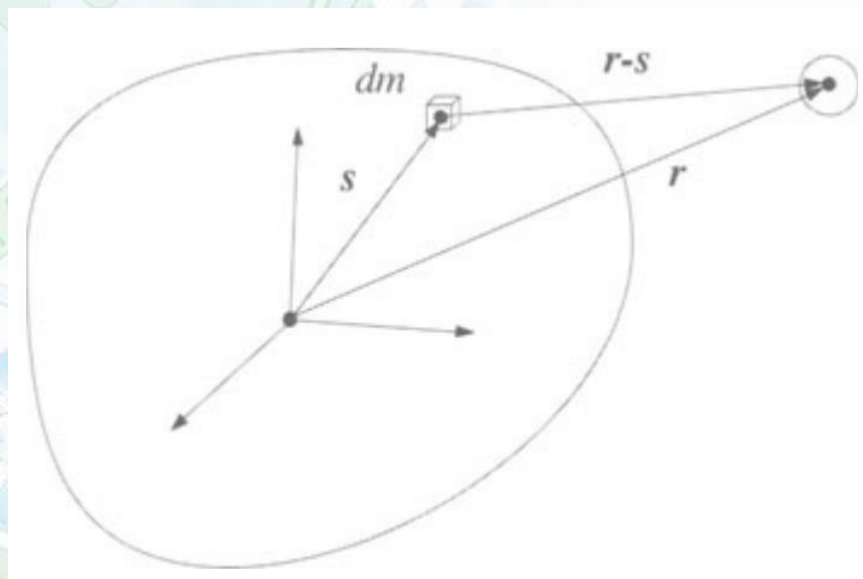
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r \cos \varphi \cos \lambda \\ r \cos \varphi \sin \lambda \\ r \sin \varphi \end{pmatrix}, \quad \begin{pmatrix} r \\ \varphi \\ \lambda \end{pmatrix} = \begin{pmatrix} \sqrt{x^2 + y^2 + z^2} \\ \tan^{-1}(z / \sqrt{x^2 + y^2}) \\ \tan^{-1}(y / x) \end{pmatrix}$$

$$\frac{\partial(r, \varphi, \lambda)}{\partial(x, y, z)} = \begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} & \frac{\partial r}{\partial z} \\ \frac{\partial \varphi}{\partial x} & \frac{\partial \varphi}{\partial y} & \frac{\partial \varphi}{\partial z} \\ \frac{\partial \lambda}{\partial x} & \frac{\partial \lambda}{\partial y} & \frac{\partial \lambda}{\partial z} \end{pmatrix} = \begin{pmatrix} \cos \varphi \cos \lambda & \cos \varphi \sin \lambda & \sin \varphi \\ -\frac{1}{r} \sin \varphi \cos \lambda & -\frac{1}{r} \sin \varphi \sin \lambda & \frac{1}{r} \cos \varphi \\ -\frac{1}{r \cos \varphi} \sin \lambda & \frac{1}{r \cos \varphi} \cos \lambda & 0 \end{pmatrix}$$

等价计算方法

$$C_{nm} = \frac{2 - \delta_{0m}}{M_{\oplus}} \frac{(n-m)!}{(n+m)!} \int \frac{s^n}{R_{\oplus}^n} P_{nm}(\sin \phi') \cos(m\lambda') \rho(s) d^3s$$

$$S_{nm} = \frac{2 - \delta_{0m}}{M_{\oplus}} \frac{(n-m)!}{(n+m)!} \int \frac{s^n}{R_{\oplus}^n} P_{nm}(\sin \phi') \sin(m\lambda') \rho(s) d^3s$$



等价计算方法

$$U = \frac{GM_{\oplus}}{R_{\oplus}} \sum_{n=0}^{\infty} \sum_{m=0}^n (C_{nm} V_{nm} + S_{nm} W_{nm})$$

$$V_{nm} = \left(\frac{R_{\oplus}}{r}\right)^{n+1} \cdot P_{nm}(\sin \phi) \cdot \cos m\lambda$$

$$W_{nm} = \left(\frac{R_{\oplus}}{r}\right)^{n+1} \cdot P_{nm}(\sin \phi) \cdot \sin m\lambda$$

$$V_{mm} = (2m - 1) \left\{ \frac{xR_{\oplus}}{r^2} V_{m-1,m-1} - \frac{yR_{\oplus}}{r^2} W_{m-1,m-1} \right\}$$

$$W_{mm} = (2m - 1) \left\{ \frac{xR_{\oplus}}{r^2} W_{m-1,m-1} + \frac{yR_{\oplus}}{r^2} V_{m-1,m-1} \right\}$$

$$V_{nm} = \left(\frac{2n-1}{n-m}\right) \cdot \frac{zR_{\oplus}}{r^2} \cdot V_{n-1,m} - \left(\frac{n+m-1}{n-m}\right) \cdot \frac{R_{\oplus}^2}{r^2} \cdot V_{n-2,m}$$

$$W_{nm} = \left(\frac{2n-1}{n-m}\right) \cdot \frac{zR_{\oplus}}{r^2} \cdot W_{n-1,m} - \left(\frac{n+m-1}{n-m}\right) \cdot \frac{R_{\oplus}^2}{r^2} \cdot W_{n-2,m}$$

等价计算方法

$$\ddot{x} = \sum_{n,m} \ddot{x}_{nm} \quad , \quad \ddot{y} = \sum_{n,m} \ddot{y}_{nm} \quad , \quad \ddot{z} = \sum_{n,m} \ddot{z}_{nm}$$

$$\ddot{x}_{nm} \stackrel{(m=0)}{=} \frac{GM}{R_{\oplus}^2} \cdot \left\{ -C_{n0} V_{n+1,1} \right\}$$

$$\begin{aligned} \stackrel{(m>0)}{=} \frac{GM}{R_{\oplus}^2} \cdot \frac{1}{2} \cdot \left\{ (-C_{nm} V_{n+1,m+1} - S_{nm} W_{n+1,m+1}) \right. \\ \left. + \frac{(n-m+2)!}{(n-m)!} \cdot (+C_{nm} V_{n+1,m-1} + S_{nm} W_{n+1,m-1}) \right\} \end{aligned}$$

$$\ddot{y}_{nm} \stackrel{(m=0)}{=} \frac{GM}{R_{\oplus}^2} \cdot \left\{ -C_{n0} W_{n+1,1} \right\}$$

$$\begin{aligned} \stackrel{(m>0)}{=} \frac{GM}{R_{\oplus}^2} \cdot \frac{1}{2} \cdot \left\{ (-C_{nm} \cdot W_{n+1,m+1} + S_{nm} \cdot V_{n+1,m+1}) \right. \\ \left. + \frac{(n-m+2)!}{(n-m)!} \cdot (-C_{nm} W_{n+1,m-1} + S_{nm} V_{n+1,m-1}) \right\} \end{aligned}$$

$$\ddot{z}_{nm} = \frac{GM}{R_{\oplus}^2} \cdot \left\{ (n-m+1) \cdot (-C_{nm} V_{n+1,m} - S_{nm} W_{n+1,m}) \right\} \cdot$$

...

...



V_{nn}, W_{nn}

V_{00}, W_{00}

V_{10}, W_{10}

V_{20}, W_{20}

⋮

V_{n0}, W_{n0}

V_{11}, W_{11}

V_{21}, W_{21}

⋮

V_{n1}, W_{n1}

V_{22}, W_{22}

⋮

V_{n2}, W_{n2}

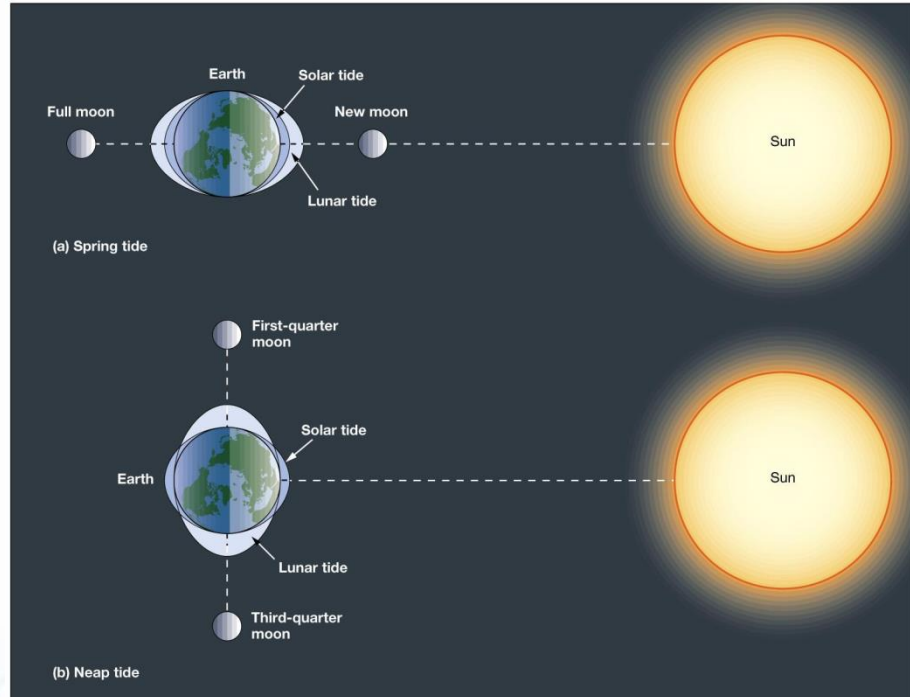
⋮

...

V_{nn}, W_{nn}

固体潮汐

地球并非刚体，由于日月引力的影响
 ⇒ 地球弹性形变 ⇒ 固体潮
 ⇒ 地球体积和密度分布之改变
 ⇒ 引力位的变化 ⇒ 固体潮摄动。



$$\Delta \bar{C}_{lm} - i\Delta \bar{S}_{lm} = \frac{k_{lm}}{2l+1} \sum_{j=2}^3 \frac{GM_j}{GM_E} \left(\frac{a_E}{r_j} \right)^{l+1} \bar{P}_{lm}(\sin \phi_j) e^{-im\lambda_j}$$

$$\begin{cases} \Delta \bar{C}_{lm} \\ \Delta \bar{S}_{lm} \end{cases}_{st} = \frac{k_{lm}}{2l+1} \sum_{j=2}^3 \left(\frac{GM_j}{GM_E} \right) \left(\frac{a_E}{r_j} \right)^{l+1} \bar{P}_{lm} \sin(\varphi_j) \begin{cases} \cos m\lambda_j \\ \sin m\lambda_j \end{cases}$$

$$\Delta \bar{S}_{l0} = 0$$

固体潮汐改正步骤

固体潮模型以Wahr模型为基础，固体潮引起发生变化。Wahr固体潮模型是弹性、椭球成层，旋转及自引力地球模型，且考虑到液核的动力学影响，因而不能用一组简单的Love数来表达出地面点的运动，Wahr模型对不同的分潮波（主要是全日波）将有不同的勒夫数。Wahr模型中Love数是随分潮波频率不同而不同。

$$\left\{ \begin{aligned} (\Delta\bar{C}_{20})_{DT_1} &= \frac{1}{\sqrt{5}} k_2 \sum_{j=1}^2 \frac{GM_j}{GE} \left(\frac{a_e}{r_j} \right)^3 P_{20}(\sin \phi_j) - \langle \Delta\bar{C}_{20} \rangle \\ (\Delta\bar{C}_{21})_{DT_1} + i(\Delta\bar{S}_{21})_{DT_1} &= \frac{1}{3} \sqrt{\frac{3}{5}} k_2 \sum_{j=1}^2 \frac{GM_j}{GE} \left(\frac{a_e}{r_j} \right)^3 P_{21}(\sin \phi_j) e^{i\lambda_j} \\ (\Delta\bar{C}_{22})_{DT_1} + i(\Delta\bar{S}_{22})_{DT_1} &= \frac{1}{12} \sqrt{\frac{12}{5}} k_2 \sum_{j=1}^2 \frac{GM_j}{GE} \left(\frac{a_e}{r_j} \right)^3 P_{22}(\sin \phi_j) e^{i2\lambda_j} \end{aligned} \right.$$

$$\left\{ \begin{aligned} (\Delta\bar{C}_{21})_{DT_2} + i(\Delta\bar{S}_{21})_{DT_2} &= \sum_{S(2,1)} A_1 \delta K_S H_S (\sin \theta_S + i \cos \theta_S) \\ (\Delta\bar{C}_{22})_{DT_2} + i(\Delta\bar{S}_{22})_{DT_2} &= \sum_{S(2,2)} A_2 \delta K_S H_S (\cos \theta_S - i \sin \theta_S) \end{aligned} \right.$$

海潮摄动

日月引力 \Rightarrow 海潮 \Rightarrow $\left\{ \begin{array}{l} \text{海水负荷变化} \\ \text{负荷变化引起形变} \end{array} \right\} \Rightarrow$ 引力位变化

海潮模型以Schwiderski (1983)模型为基础的，它的动力学效应也可以通过引力场系数的修正来体现的。

$$\Delta \bar{C}_{lm} - i\Delta \bar{S}_{lm} = F_{lm} \sum_{s(l,m)} \sum_{+} (C_{snm}^{\pm} \mp iS_{snm}^{\pm}) e^{\pm i\theta_s}$$

$$F_{lm} = \frac{4\pi a_E^2 \rho_w}{M_E} \sqrt{\frac{(l+m)!}{(2-\delta_{0m})(2l+1)(l-m)!}} \left(\frac{1+k_l'}{2l+1} \right)$$

$$\left\{ \begin{array}{l} \Delta \bar{C}_{lm} = F_{lm} \sum_{s(l,m)} ((C_{slm}^+ + C_{slm}^-) \cos \theta_s + (S_{slm}^+ + S_{slm}^-) \sin \theta_s) \\ \Delta \bar{S}_{lm} = F_{lm} \sum_{s(l,m)} ((S_{slm}^+ - S_{slm}^-) \cos \theta_s - (C_{slm}^+ - C_{slm}^-) \sin \theta_s) \end{array} \right.$$



大气潮

大气潮起因——引力源、热源（主要），这也是主大气潮具有半太阳日的周期并比月潮大15倍左右的原因。据Lambeck等人（1974年）研究，大气潮中仅 S_2 波是主要的，而且 S_2 大气潮汐摄动的效应相当于固体潮效应的2.5%。

$$(\Delta \bar{C}_{nm})_{AT} = \sum_{\mu(n,m)} F_{nm} \left[(C_{\mu nm}^{A+} + C_{\mu nm}^{A-}) \cos(\bar{n}_{\mu} \cdot \bar{\beta}) + (S_{\mu nm}^{A+} + S_{\mu nm}^{A-}) \sin(\bar{n}_{\mu} \cdot \bar{\beta}) \right]$$

$$(\Delta \bar{S}_{nm})_{AT} = \sum_{\mu(n,m)} F_{nm} \left[(S_{\mu nm}^{A+} + S_{\mu nm}^{A-}) \cos(\bar{n}_{\mu} \cdot \bar{\beta}) - (C_{\mu nm}^{A+} - C_{\mu nm}^{A-}) \sin(\bar{n}_{\mu} \cdot \bar{\beta}) \right]$$

$$\bar{C}_{S_2 22}^{A+} = 0.344 mb \quad C_{S_2 22}^{A+} = \frac{\bar{C}_{S_2 22}^{A+} \sin \varepsilon_{S_2 22}^{A+}}{g \rho_w}$$

$$\varepsilon_{S_2 22}^{A+} = 158^{\circ} \quad S_{S_2 22}^{A+} = \frac{\bar{C}_{S_2 22}^{A+} \cos \varepsilon_{S_2 22}^{A+}}{g \rho_w}$$

地球自转形变附加摄动

地球自转离心力 \Rightarrow 形变 \Rightarrow 地球体积和密度分布的改变将引入一个附加位 \Rightarrow 对卫星运动产生附加摄动

$$\left\{ \begin{array}{l} (\Delta\bar{C}_{20})_{R0} = -\frac{1}{\sqrt{5}} \frac{2a_e^3}{3GE} k_2 m_3 \Omega^2 \\ (\Delta\bar{C}_{21})_{R0} = -\frac{1}{\sqrt{15}} \frac{a_e^3}{GE} k_2 m_1 \Omega^2 \\ (\Delta\bar{S}_{21})_{R0} = -\frac{1}{\sqrt{15}} \frac{a_e^3}{GE} k_2 m_2 \Omega^2 \end{array} \right.$$

$$m_1 = x_p, \quad m_2 = -y_p, \quad m_3 = -\frac{D}{86400000}$$

非球形引力位与潮汐总加速度

$$U = \frac{GE}{r} \sum_{n=2}^N \sum_{m=0}^n \left(\frac{a_e}{r} \right)^n \bar{P}_{nm}(\sin \phi) (\bar{C}_{nm}^* \cos m\lambda + \bar{S}_{nm}^* \sin m\lambda)$$

$$\begin{cases} \bar{C}_{nm}^* = \bar{C}_{nm} + (\Delta \bar{C}_{nm})_{DT} + (\Delta \bar{C}_{nm})_{OT} + (\Delta \bar{C}_{nm})_{AT} + (\Delta \bar{C}_{nm})_{R0} \\ \bar{S}_{nm}^* = \bar{S}_{nm} + (\Delta \bar{S}_{nm})_{DT} + (\Delta \bar{S}_{nm})_{OT} + (\Delta \bar{S}_{nm})_{AT} + (\Delta \bar{S}_{nm})_{R0} \end{cases}$$

$$\vec{A}_{NS} + \vec{A}_{DT} + \vec{A}_{OT} + \vec{A}_{AT} + \vec{A}_{R0} = (\mathbf{HG})^T \left(\frac{\partial U}{\partial \vec{r}(x, y, z)} \right)^T$$

HG矩阵见时间与空间部分

天球与地球坐标系转换回顾

$$\mathbf{r}_{\text{GCRS}} = \mathbf{B}\mathbf{P}(t)\mathbf{N}(t)\mathbf{S}(t)\mathbf{W}(t)\mathbf{r}_{\text{ITRS}}$$

$$\mathbf{r}_{\text{ITRS}} = [\mathbf{W}(t)]^T [\mathbf{S}(t)]^T [\mathbf{N}(t)]^T [\mathbf{P}(t)]^T [\mathbf{B}]^T \mathbf{r}_{\text{GCRS}}$$

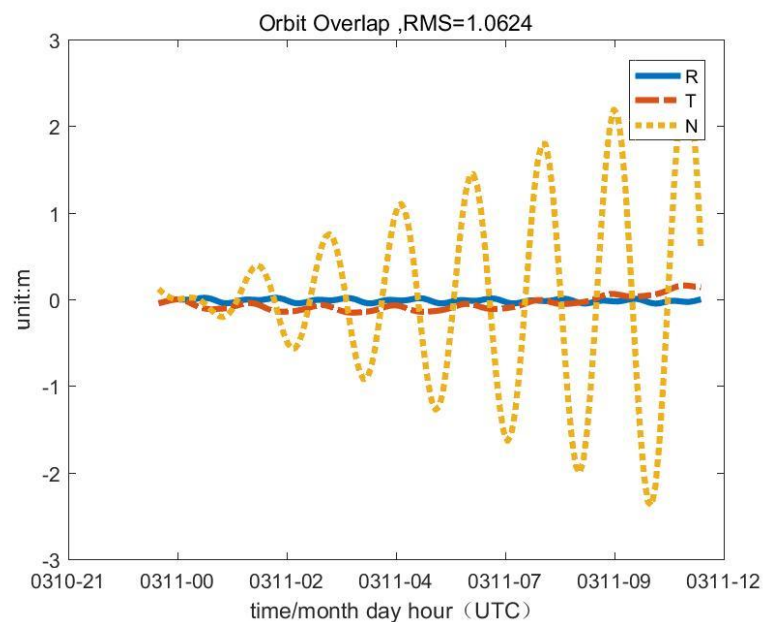
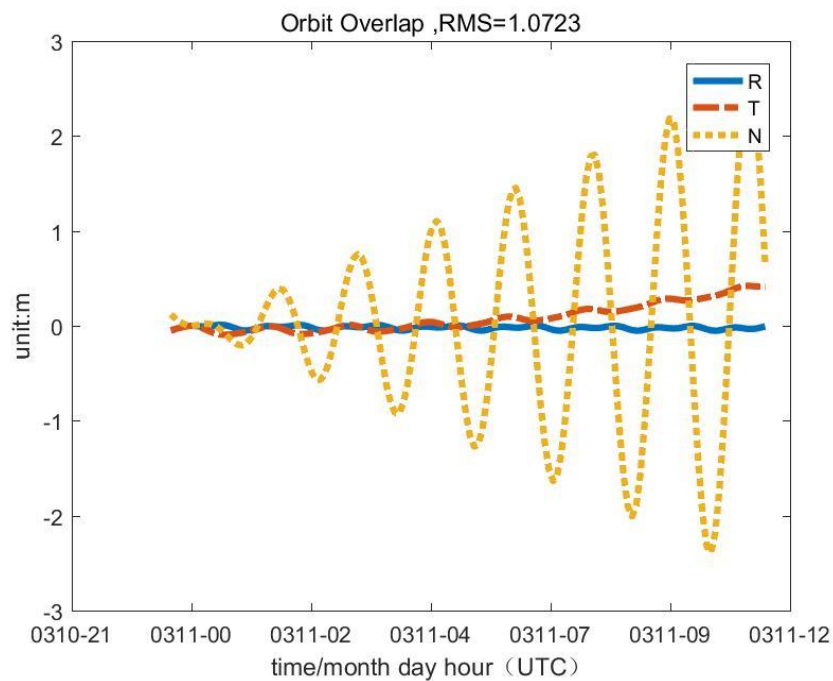
$$\begin{aligned} \mathbf{v}_{\text{ITRS}} &= [\mathbf{W}]^T \left\{ [\mathbf{S}]^T [\mathbf{BPN}]^T \mathbf{v}_{\text{GCRS}} - \boldsymbol{\omega}_E \times \mathbf{r}_{\text{ECEFw/oPM}} \right\} \\ &= [\mathbf{W}]^T [\mathbf{S}]^T [\mathbf{BPN}]^T \mathbf{v}_{\text{GCRS}} + [\mathbf{W}]^T [\mathbf{S}']^T [\mathbf{BPN}]^T \mathbf{r}_{\text{GCRS}} \end{aligned}$$

$$\begin{aligned} \mathbf{v}_{\text{GCRS}} &= [\mathbf{BPN}] [\mathbf{S}] \left\{ [\mathbf{W}] \mathbf{v}_{\text{ITRS}} + \boldsymbol{\omega}_E \times \mathbf{r}_{\text{ECEFw/oPM}} \right\} \\ &= [\mathbf{BPN}] [\mathbf{S}] [\mathbf{W}] \mathbf{v}_{\text{ITRS}} + [\mathbf{BPN}] [\mathbf{S}'] [\mathbf{W}] \mathbf{r}_{\text{GCRS}} \end{aligned}$$

两个典型引力场模型

- EGM2008地球重力场模型是由NGA发布的全球超高阶重力场模型，它以PGM2007B为参考，综合利用GRACE卫星重力数据、全球5'×5'重力异常数据、TOPEX卫星测高数据、地形数据、地面重力数据。该地球重力场模型研制周期为4年，球谐系数的阶扩展至2190，阶次完全至2159，经过大量的测试与评估结果表明，2190阶的EGM2008地球重力场模型比其他模型的精度有了大幅度的提高，是迄今为止分辨率最高、精度最好、阶次最多的全球重力场模型之一。
- EIGEN-6C4重力场模型是由GFZ在2014年11月发布的最新2190阶次重力场模型，成为继EIGEN-6C、EIGEN-6C2、EIGEN-6C3sata后，发布的又一超高阶重力场模型。该模型采用卫星重力数据（GOCE、GRACE、SLR）、地面重力数据与卫星测高数据解算而成，最高阶可达2190（前50阶含有时变参数）。与之前发布的模型相比，该模型在中长波段的精度有较大的提升，也是目前较优的一个超高阶地球重力场模型。

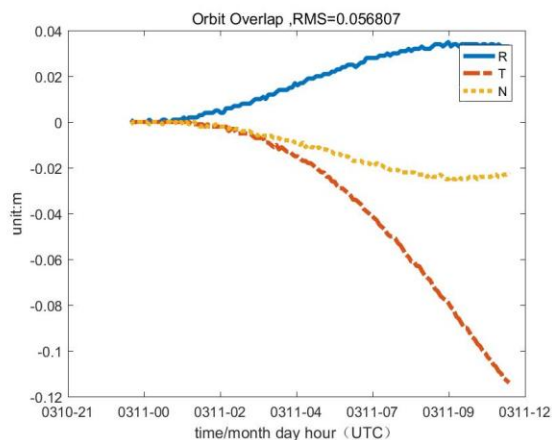
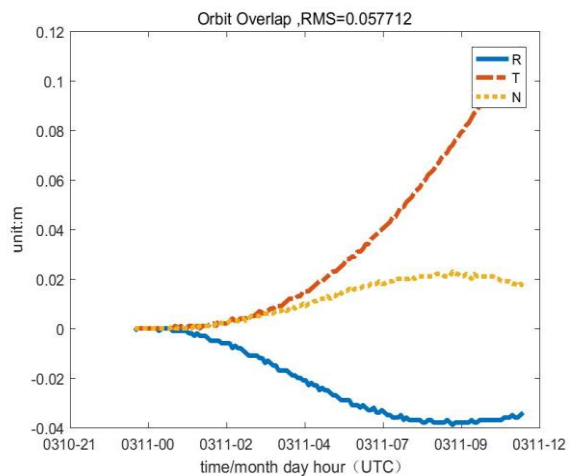
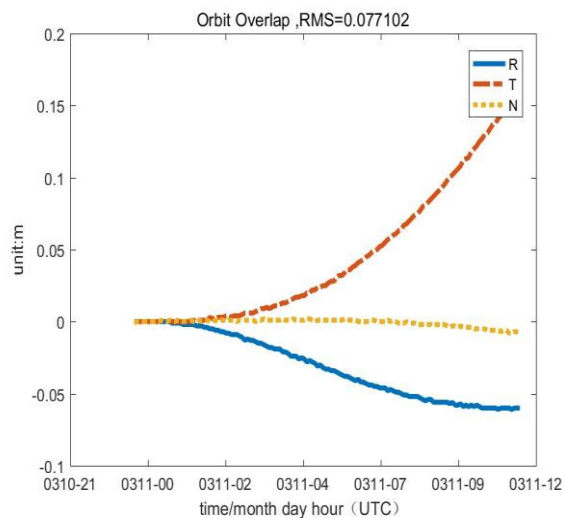
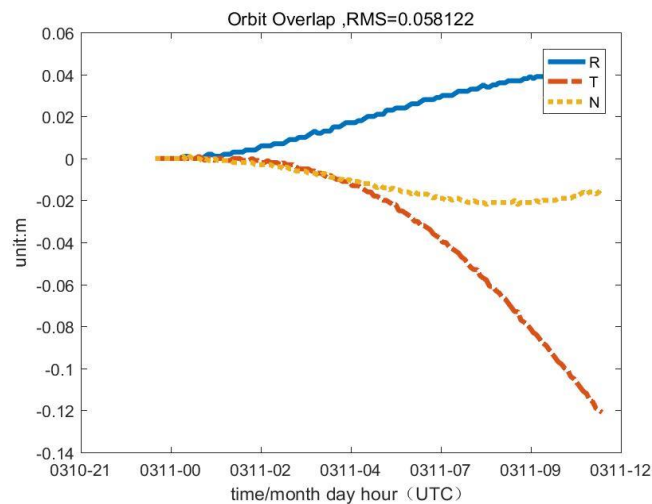
固体潮对轨道影响LEO (1天)



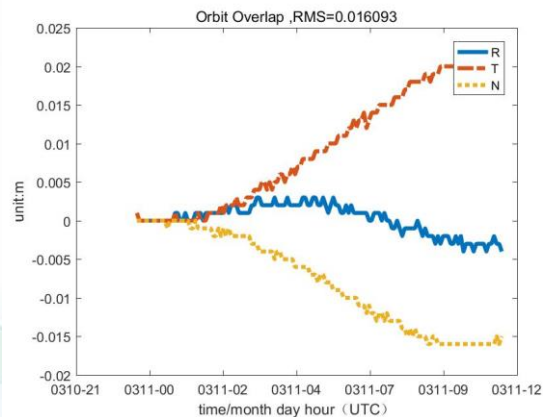
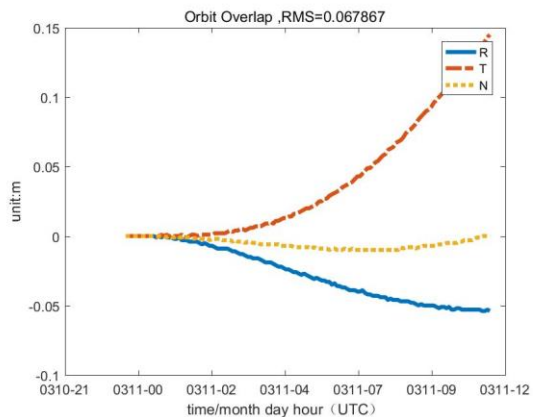
GRACE_A,GRACE_B

固体潮对IGSO轨道影响 (1天)

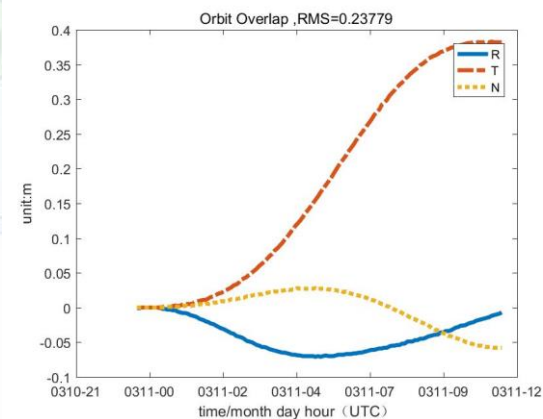
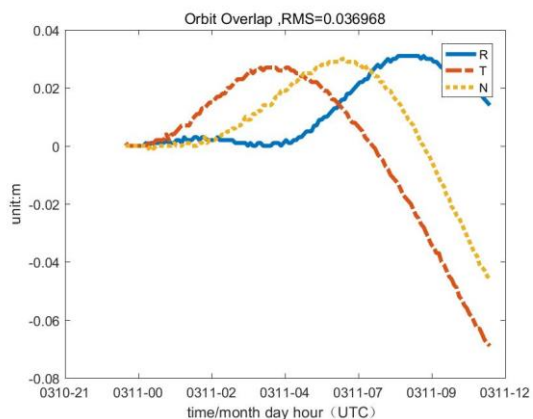
C06,C07,C08,C09



固体潮对GEO与MEO影响 (1天)



C01,C05



C11,C12

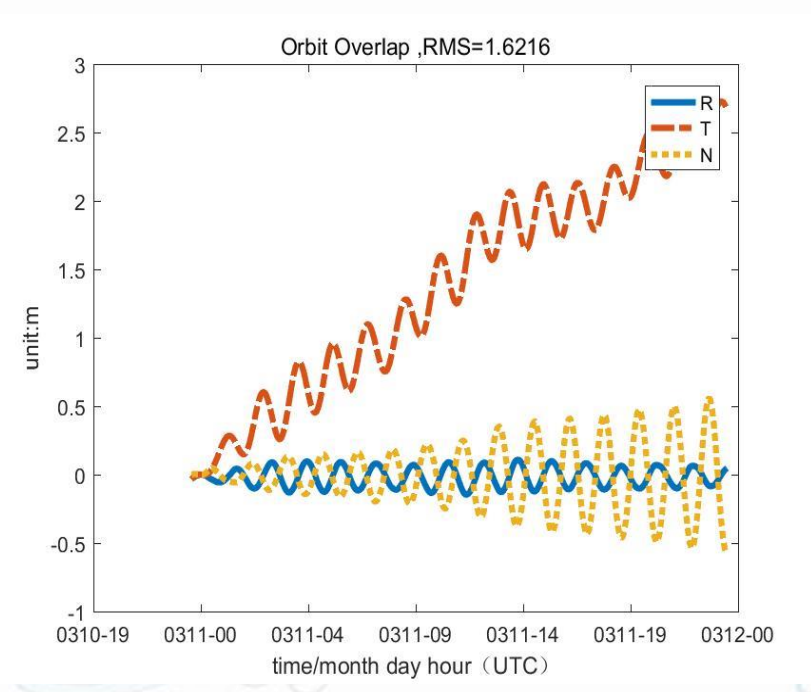
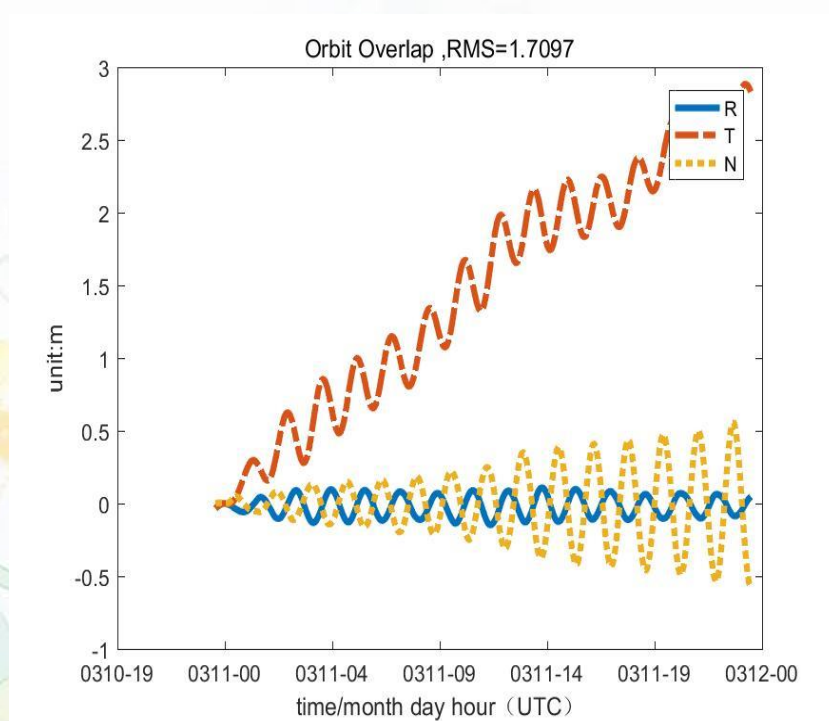
固体潮对轨道影响积分一天统计

Nosolid	N(m)	T(m)	R(m)	3D(m)
GEOC01	0.0354	0.1987	0.0152	0.2024
GRACE_A	0.0322	0.9675	2.2192	2.4212
GRACE_B	0.0296	0.6715	2.2197	2.3192
GEOC05	0.003	0.0226	0.0158	0.0278
IGSOC06	0.0262	0.1607	0.0266	0.165
IGSOC07	0.0415	0.2264	0.0068	0.2303
IGSOC08	0.023	0.1267	0.0257	0.1313
IGSOC09	0.0206	0.1332	0.0266	0.1374
IGSOC10	0.0224	0.1425	0.0068	0.1444
MEOC11	0.0183	0.0575	0.0443	0.0749
MEOC12	0.046	0.4205	0.0498	0.4259
US_HEO2	0.0131	0.3377	0.1288	0.3617

固体潮对轨道影响积分30天统计

Nosolid	N(m)	T(m)	R(m)	3D(m)
GEOC01	0.0441	7.0264	0.2989	7.0329
GRACE_A	0.363	190.7456	26.8148	192.6215
GRACE_B	0.355	182.1717	26.7715	184.1287
IGSOC07	0.2258	26.6251	0.7151	26.6357
IGSOC08	0.2539	26.9387	0.8266	26.9525
IGSOC09	0.2571	26.8067	0.4866	26.8124
MEOC11	0.0445	10.7296	2.2248	10.9579
MEOC12	0.0347	4.1011	2.2361	4.6712
US_HEO2	0.891	12.9852	9.1938	15.9354

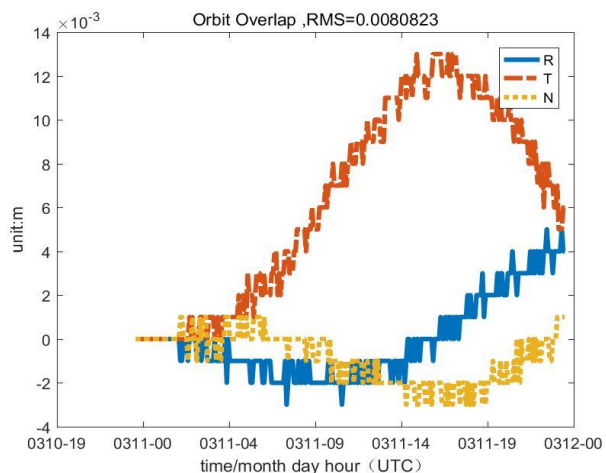
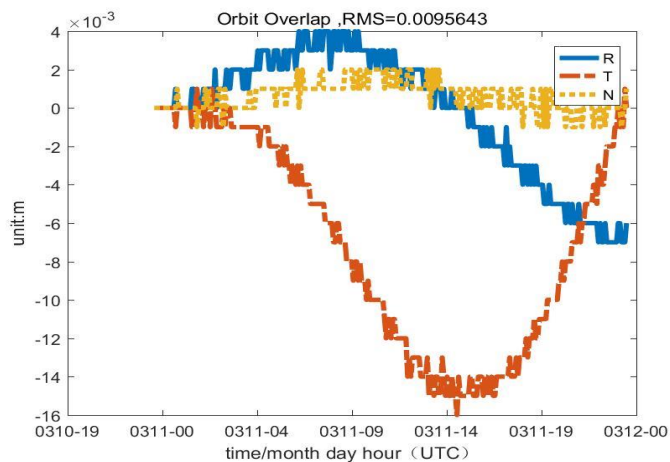
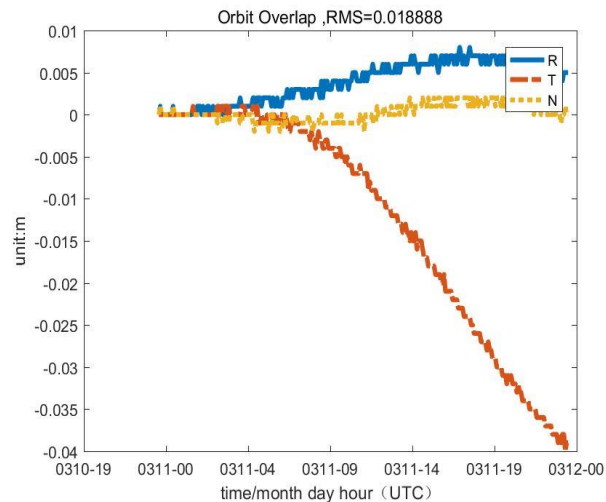
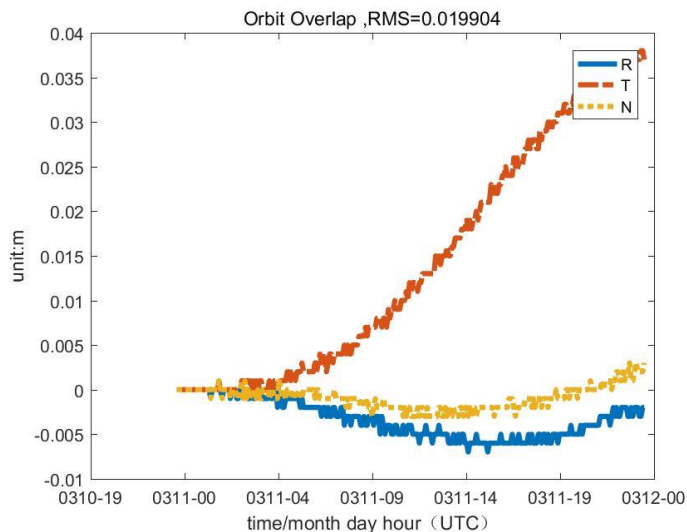
海潮对LEO轨道影响 (1天)



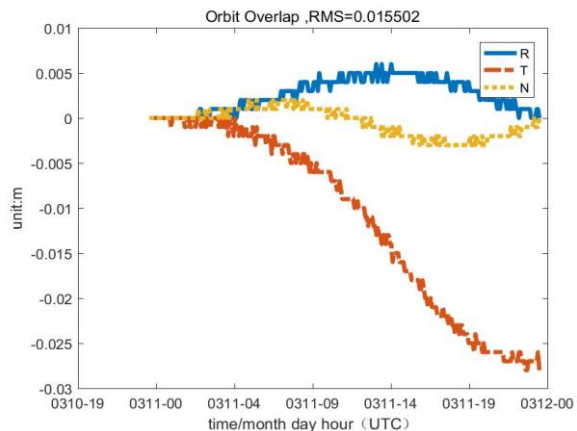
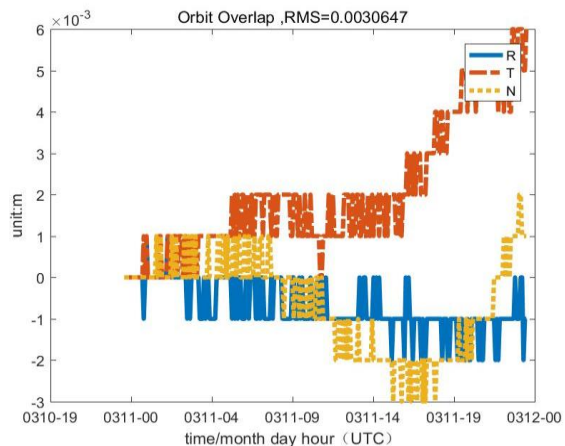
GRACE_A,GRACE_B

海潮对IGSO轨道影响 (1天)

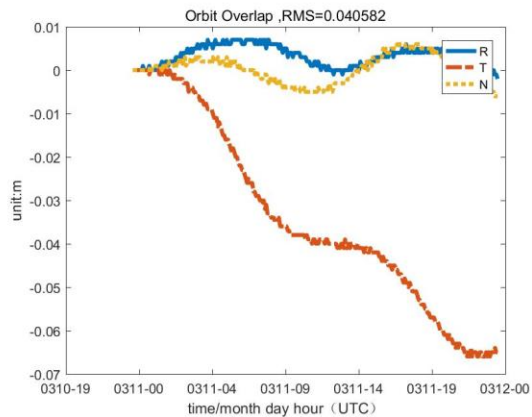
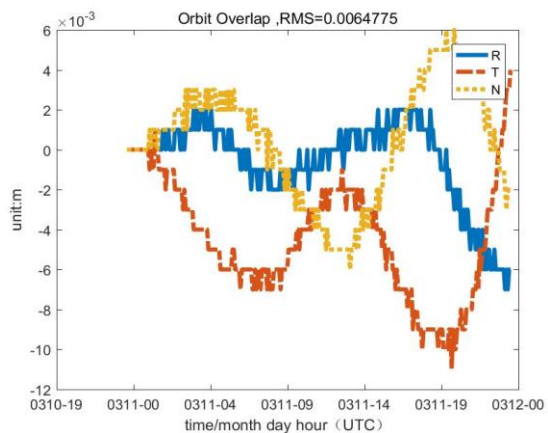
C06,C07,C08,C09



海潮对GEO与MEO轨道影响 (1天)



C01,C05



C11,C12

海潮对卫星轨道影响统计（1天）

Noocan_1 day	N(m)	T(m)	R(m)	3D(m)
GEOC01	0.0009	0.0026	0.0013	0.0031
GRACE_A	0.0717	1.6917	0.2367	1.7097
GRACE_B	0.0709	1.6026	0.2371	1.6216
GEOC05	0.0031	0.0151	0.0016	0.0155
MEOC06	0.0039	0.0195	0.0016	0.0199
MEOC07	0.0045	0.0183	0.001	0.0189
MEOC08	0.0033	0.0089	0.001	0.0096
MEOC09	0.0018	0.0077	0.0014	0.0081
MEOC10	0.0044	0.0218	0.0014	0.0223
IGSOC11	0.0022	0.0054	0.0029	0.0065
IGSOC12	0.0039	0.0403	0.0032	0.0406

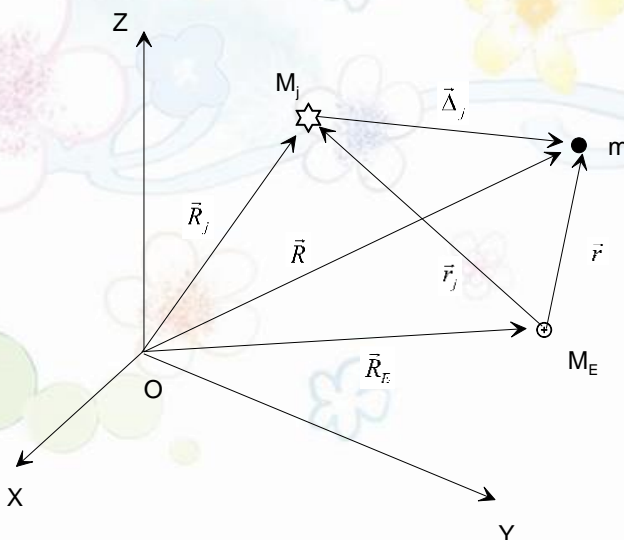
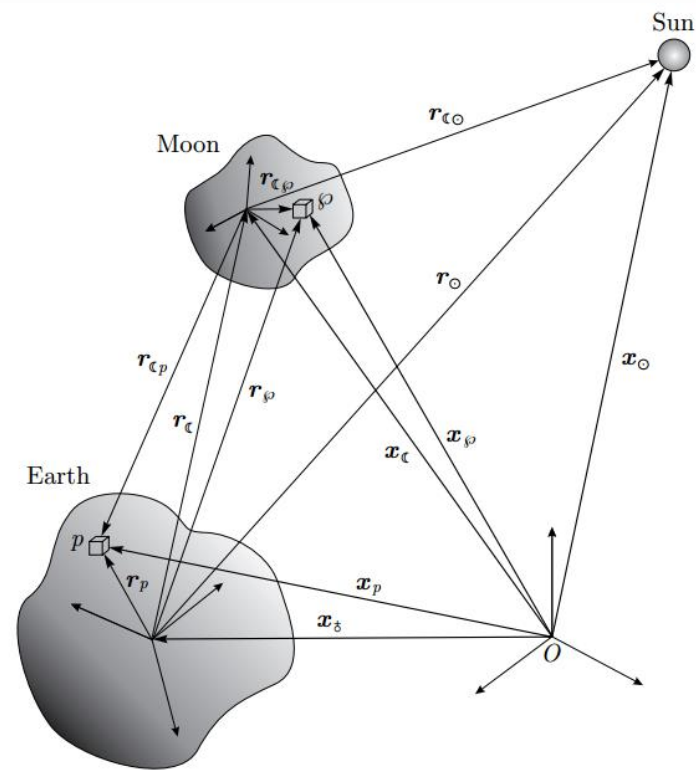
海潮对卫星轨道影响统计（30天）

Nooccean_30days	N(m)	T(m)	R(m)	3D(m)
GEOC01	0.0064	0.0364	0.0243	0.0442
GRACE_A	0.2064	19.1985	1.7013	19.2748
GRACE_B	0.206	16.4789	1.7031	16.568
GEOC05	0.0199	0.204	0.0251	0.2065
MEOC06	0.0493	3.3428	0.0128	3.3432
MEOC07	0.0205	2.6312	0.01	2.6313
MEOC08	0.0567	5.8022	0.0105	5.8025
MEOC09	0.0627	4.8813	0.0136	4.8817
MEOC10	0.0272	1.1867	0.0117	1.1871
IGSOC11	0.014	1.1283	0.0532	1.1297
IGSOC12	0.0161	1.3155	0.0525	1.3166

第三体引力

$$\begin{aligned} \vec{r} &= -\frac{GM_E}{r^2} \cdot \frac{\vec{r}}{r} - \frac{GM_j}{\Delta_j^2} \cdot \frac{\vec{\Delta}_j}{\Delta_j} - \frac{GM_j}{r_j^2} \cdot \frac{\vec{r}_j}{r_j} \\ &= -\frac{GM_E}{r^2} \cdot \frac{\vec{r}}{r} - GM_j \left(\frac{\vec{r}_j}{r_j^3} + \frac{\vec{\Delta}_j}{\Delta_j^3} \right) \end{aligned}$$

$$\ddot{\vec{r}}_N = -\sum_{j=1}^N GM_j \left(\frac{\vec{r}_j}{r_j^3} + \frac{\vec{\Delta}_j}{\Delta_j^3} \right)$$



JPL行星历表

- ▶ JPL 星历是由美国喷气推进实验室研制，目前是为太空导航，行星探测以及精密天文观测的分析和归算提供精密数据，目前JPL 的主要星历有DE200、DE403、DE405、DE430。

如DE405，覆盖了从1600年到2170年大约600年时间段。

所有星历都基于各自运动方程进行严格数值积分。除了月球、行星、和太阳的点质量相互作用外，部分小行星的摄动和运动方程的相对论后牛顿修正也要考虑。另外，日月扭矩对地球形状的影响，以及地球和太阳扭矩对月球形状的影响都精细了考虑

Mass parameters from DE421 expressed as ratios and as TDB-compatible values.

	GM_{\odot}/GM_i	$GM_i/\text{km}^3\text{s}^{-2}$
Mercury	6023597.400017	22032.090000
Venus	408523.718655	324858.592000
Earth	332946.048166	398600.436233
Moon	27068703.185436	4902.800076
Mars	3098703.590267	42828.375214
Jupiter	1047.348625	126712764.800000
Saturn	3497.901768	37940585.200000
Uranus	22902.981613	5794548.600000
Neptune	19412.237346	6836535.000000
Pluto	135836683.767599	977.000000

	GM_{\oplus}/GM_i
Earth-Moon mass ratio	81.3005690699

DE历表切比雪夫多项式逼近

	水星	金星	地月系质心	火星	木星	土星	天王星	海王星	冥王星	月球相对地心	太阳	章动
编号	1	2	3	4	5	6	7	8	9	10	11	12
l	4	1	2	1	1	1	1	1	1	8	1	4
N	12	12	15	10	9	8	8	6	6	12	15	10

$$\begin{cases} T_1(\tau) = 1 \\ T_2(\tau) = \tau \\ T_i(\tau) = 2\tau T_{i-1}(\tau) - T_{i-2}(\tau), i \geq 3 \end{cases} \quad \begin{cases} T_1'(\tau) = 0 \\ T_2'(\tau) = 1 \\ T_i'(\tau) = 2T_{i-1}(\tau) + 2\tau T_{i-1}' - T_{i-2}'(\tau), i \geq 3 \end{cases}$$

$$\tau = \frac{2(t - t_0)}{\Delta t} - 1$$

拟合出系数后，如果需要计算速度，则需要把速度量纲的分母项还原为原量纲。如，原始位置速度单位为km，拟合数据区间为。拟合出的速度分母量纲为无单位量纲。可以通过以下式子还原为 m/s量纲。

$$v = \frac{v_{fit} \times 1000}{60 \times \Delta t \times 2}$$

太阳简化解析历表

$$\left\{ \begin{array}{l} a = 149600000 \text{ km} \\ e = 0.016709 \\ i = 0.^{\circ}0000 \\ \Omega + \omega = 282.^{\circ}9400 \\ M = 359.^{\circ}5256 + 35999.^{\circ}049T \end{array} \right.$$

$$\vec{r}_{\odot} = R_x(-\varepsilon) \begin{pmatrix} r_{\odot} \cos \lambda_{\odot} \cos \beta_{\odot} \\ r_{\odot} \sin \lambda_{\odot} \cos \beta_{\odot} \\ r_{\odot} \sin \beta_{\odot} \end{pmatrix}$$

$$\varepsilon = 23.^{\circ}4329111$$

$$\left\{ \begin{array}{l} \lambda_e = \Omega + \omega + M + 6892'' \sin M + 72'' \sin 2M \\ r_e = (149.619 - 2.499 \cos M - 0.021 \cos 2M) \times 10^6 \text{ km} \end{array} \right.$$

月球简化解析历表

月球平黄经 L_0 、月球平近点角 l 、太阳平近点角 l' 、月球平升交点经度 F 、太阳平黄经和月球平黄经之间的差 D 。

$$\left\{ \begin{array}{l} L_0 = 218^\circ.31617 + 481267^\circ.88088 \cdot T - 1^\circ.3972 \cdot T \\ l = 134^\circ.96292 + 477198^\circ.86753 \cdot T \\ l' = 357^\circ.52543 + 35999^\circ.04944 \cdot T \\ F = 93^\circ.27283 + 483202^\circ.01873 \cdot T \\ D = 297^\circ.85027 + 445267^\circ.11135 \cdot T \end{array} \right.$$

月球简化历表

2000年黄道和春分点的月球黄经

$$\begin{aligned}\lambda_M = & L_0 + 22640'' \cdot \sin(l) + 729'' \sin(2l) \\ & - 4589'' \cdot \sin(l - 2D) + 2370'' \cdot \sin(2D) \\ & - 668'' \cdot \sin(l') - 412'' \cdot \sin(2F) \\ & - 212'' \cdot \sin(2l - 2D) - 206'' \cdot \sin(l + l' - 2D) \\ & + 192'' \cdot \sin(l + 2D) - 165'' \cdot \sin(l' - 2D) \\ & + 148'' \cdot \sin(l - l') - 125'' \cdot \sin(D) \\ & - 110'' \cdot \sin(l + l') - 55'' \cdot \sin(2F - 2D)\end{aligned}$$

月球纬度

$$\begin{aligned}\beta_M = & 18520'' \cdot \sin(F + \lambda - L_0) + 412'' \cdot \sin 2F + 541'' \cdot \sin l' \\ & - 526'' \cdot \sin(F - 2D) + 44'' \cdot \sin(l + F - 2D) \\ & - 31'' \cdot \sin(l' + F - 2D) - 25'' \cdot \sin(-2l + F) \\ & + 23'' \cdot \sin(l' + F - 2D) + 21'' \cdot \sin(-l + F) \\ & + 11'' \cdot \sin(-l' + F - 2D)\end{aligned}$$

月球简化历表

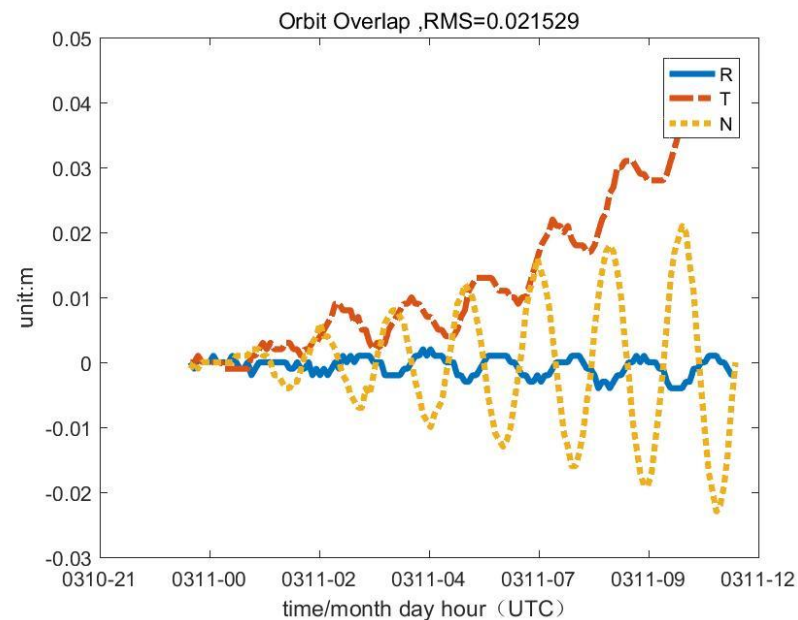
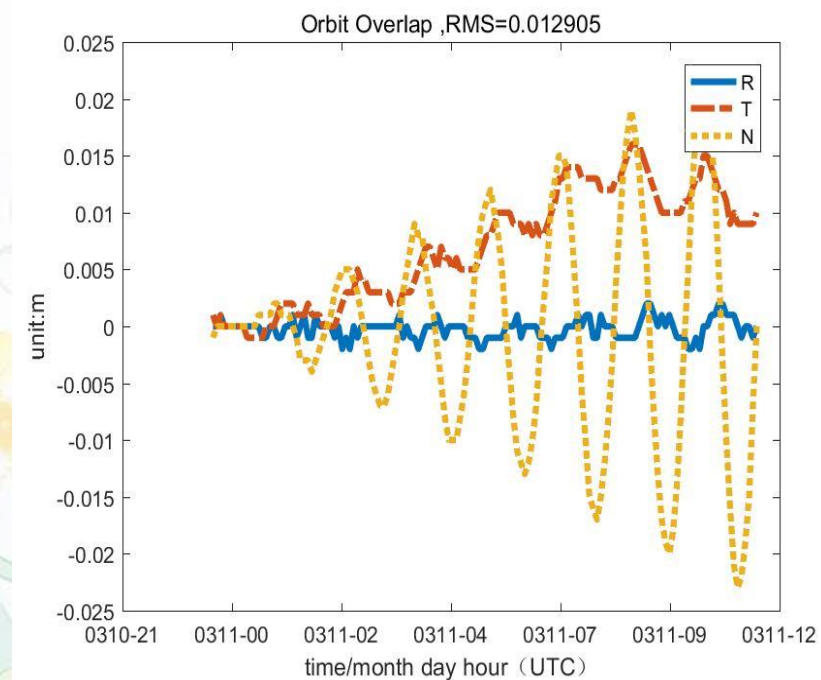
月球的地心距

$$\begin{aligned} r_M = & (35800 - 20905 \cos(l) - 3699 \cos(2D - l) \\ & - 2956 \cos(2D) - 570 \cos(2l) + 246 \cos(2l - 2D) \\ & - 205 \cos(l' - 2D) - 171 \cos(l + 2D) \\ & - 152 \cos(l + l' - 2D)) \end{aligned}$$

黄道球坐标转化为赤道笛卡尔直角坐标

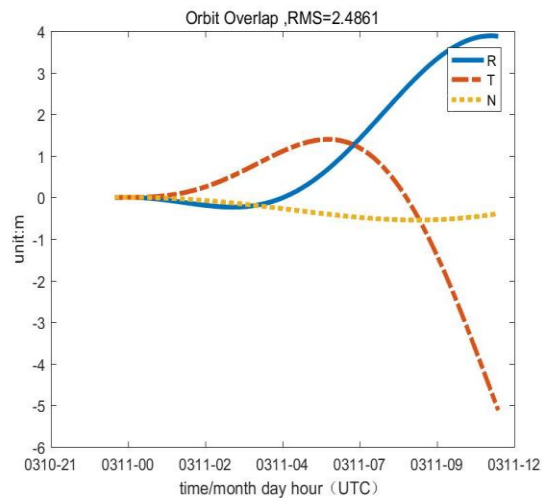
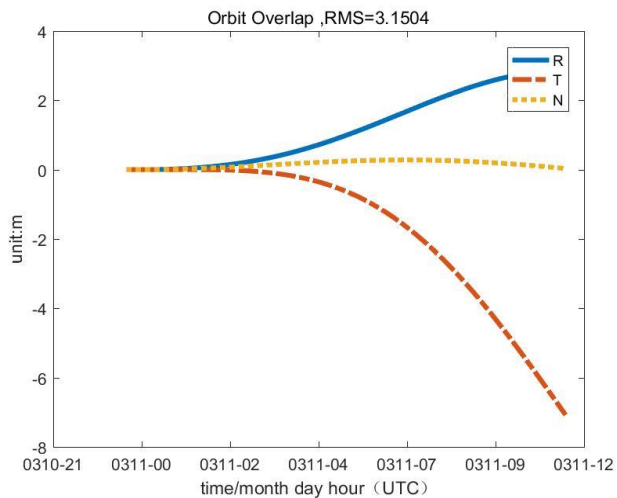
$$\vec{r}_m = R_x(-\varepsilon) \begin{pmatrix} r_M \cos \lambda_M \cos \beta_M \\ r_M \sin \lambda_M \cos \beta_M \\ r_M \sin \beta_M \end{pmatrix}$$

简化历表与数值历表比较LEO

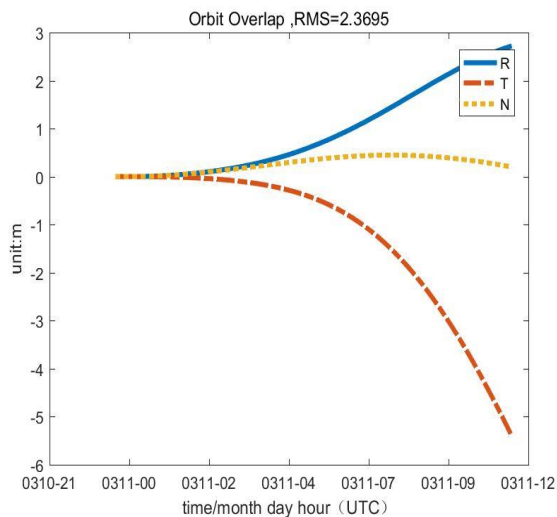
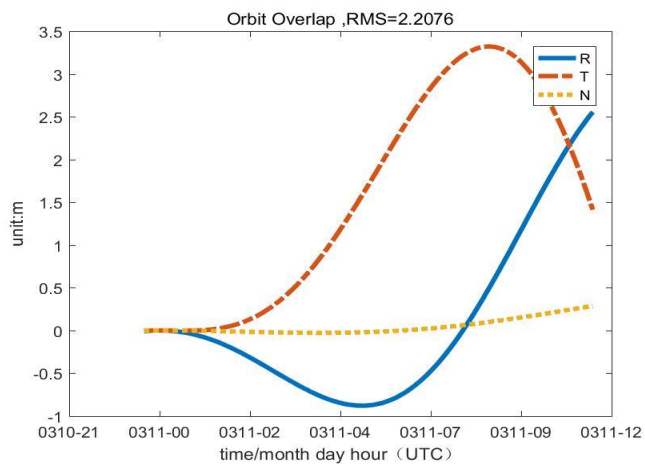


GRACE A,B 积分一天解析历表与JPL数值历表轨道比较

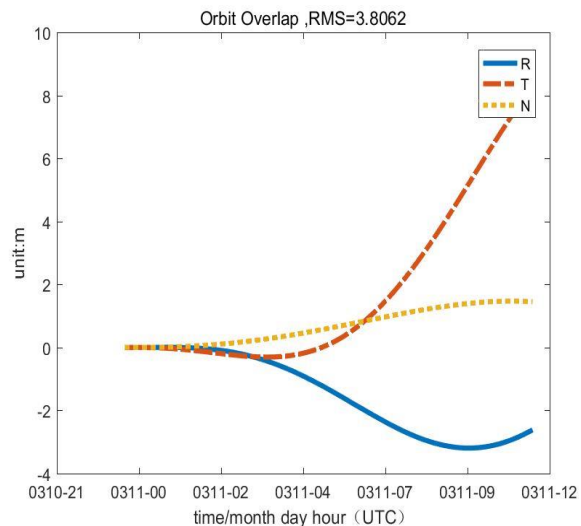
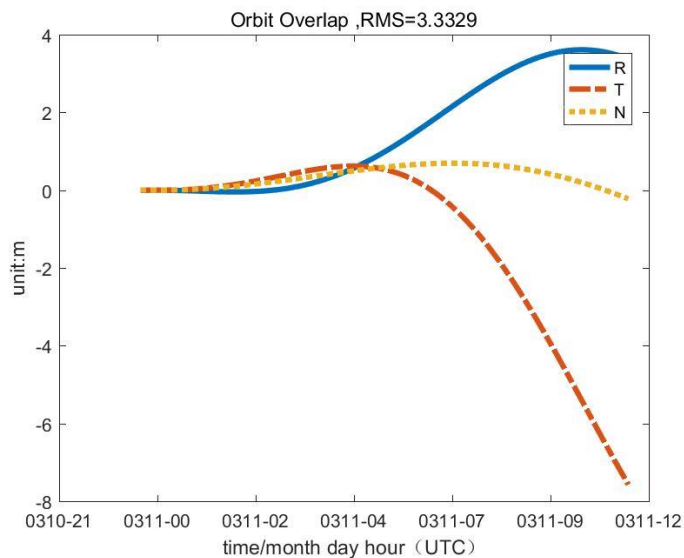
简化历表与数值历表比较IGSO



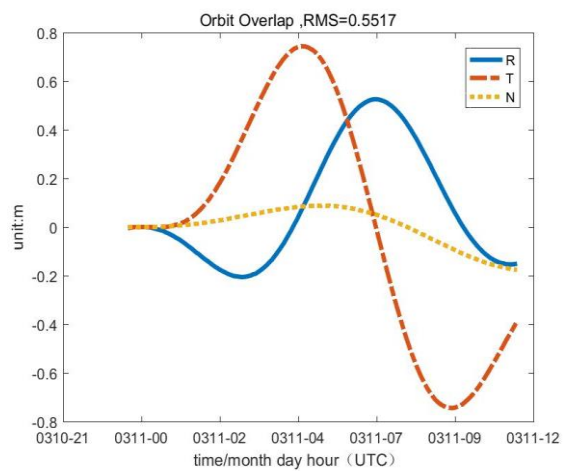
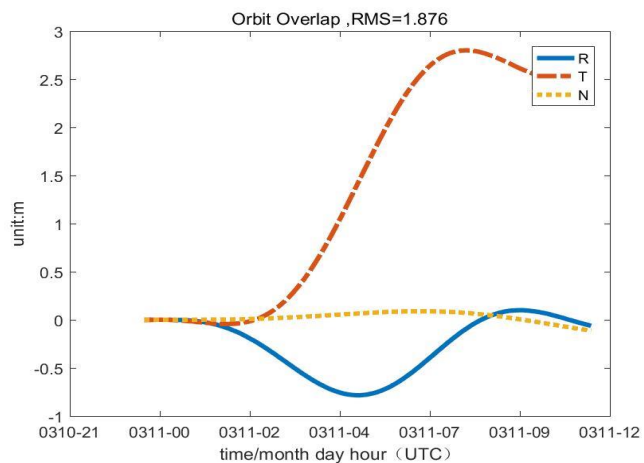
C06,C07,C08,C09
积分一天



简化历表与数值历表比较GEO\MEO

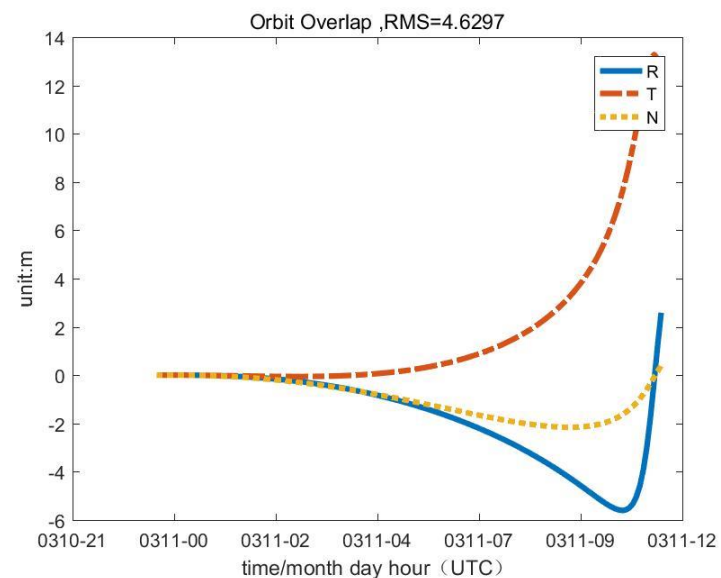
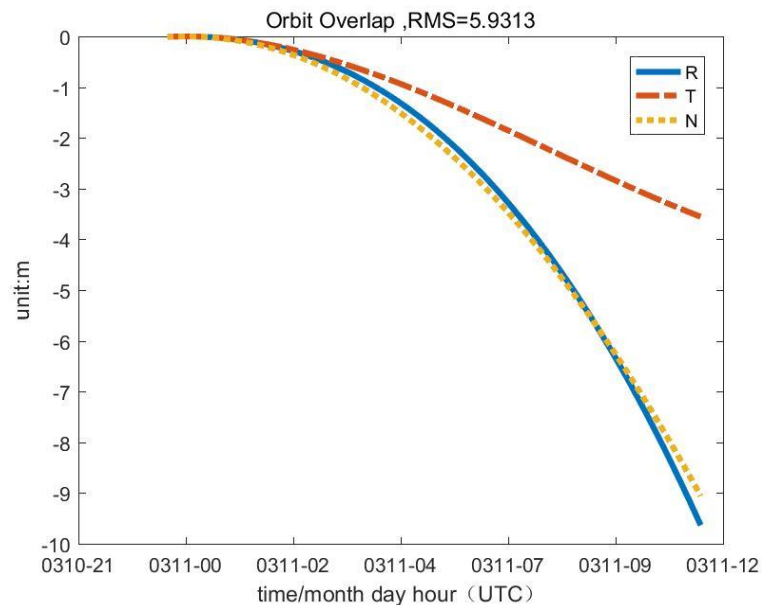


C01,C05



C11,C12

简化历表与数值历表比较HEO



积分一天轨道比较统计分析

Noocan_30days	N(m)	T(m)	R(m)	3D(m)
GEOC01	2.044921	2.596569	0.429131	3.33287
GRACE_A	0.000898	0.00857	0.009607	0.012905
GRACE_B	0.001557	0.019326	0.009359	0.021529
GEOC05	1.938766	3.158202	0.868272	3.806172
MEOC06	1.593092	2.711815	0.182177	3.150408
MEOC07	1.895083	1.568622	0.359204	2.48615
MEOC08	0.94639	1.991918	0.099781	2.207566
MEOC09	1.303387	1.955767	0.300776	2.369453
MEOC10	1.038184	2.111138	0.370861	2.381653
IGSOC11	0.394376	1.833301	0.053705	1.876009
IGSOC12	0.253381	0.484054	0.076593	0.551703
US_HEO	4.021561	1.809049	3.966737	5.931325
US_HEO2	2.499938	3.691204	1.248717	4.629684

积分30天轨道比较统计分析

Noocean_30days	N(m)	T(m)	R(m)	3D(m)
GEOC01	5.67623	211.7767	19.78732	212.7749
GRACE_A	0.02717	2.21848	0.663994	2.315876
GRACE_B	0.071325	47.6427	0.64765	47.64716
GEOC05	5.115733	34.17223	19.69858	39.77368
MEOC06	3.002702	163.2738	7.80003	163.4876
MEOC07	3.452725	206.2386	36.61507	209.4921
MEOC08	5.294057	64.64857	34.66022	73.54451
MEOC09	2.980078	171.5056	8.823836	171.7583
MEOC10	2.964932	176.6405	36.74849	180.447
IGSOC11	1.554673	84.7783	11.81714	85.61204
IGSOC12	1.502689	16.3393	11.84196	20.23519
US_HEO	1038.385	1303.135	458.5185	1728.191
US_HEO2	66.37751	89.24002	39.30728	117.9611



Q&A!