



中国科学院上海天文台

Shanghai Astronomical Observatory, Chinese Academy of Science



中国科学院大学

University of Chinese Academy of Sciences

卫星轨道确定方法

宋叶志
2020年秋季

作业邮箱: song.yz@foxmail.com

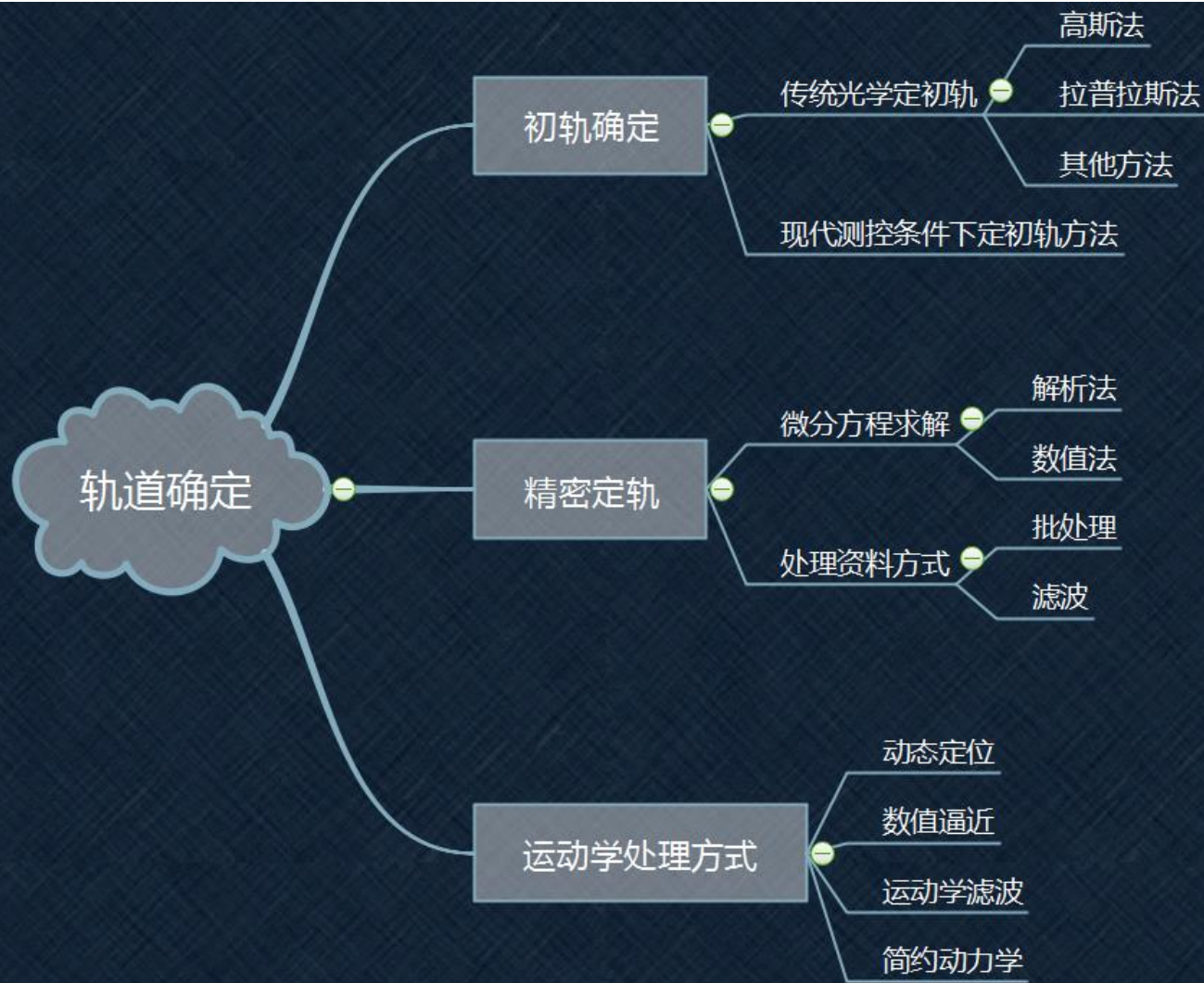
课件地址: <http://202.127.29.4/astrodynamics/course.php>

课程主要内容

- 轨道确定问题
- 初轨确定问题
- 精密定轨方法



轨道确定



改进拉普拉斯法定初轨 (一)

二体F、G级数

$$\begin{cases} \vec{r} = F \vec{r}_0 + G \dot{\vec{r}}_0 \\ \dot{\vec{r}} = F' \dot{\vec{r}}_0 + G' \vec{r}_0 \end{cases}$$

$$\begin{cases} F = 1 - \left(\frac{1}{2}u_0\right)\Delta t^2 + \left(\frac{1}{2}u_0p_0\right)\Delta t^3 + \left(\frac{1}{8}u_0q_0 - \frac{1}{12}u_0^2 - \frac{5}{8}u_0p_0^2\right)\Delta t^4 + O(\Delta t^5) \\ G = \Delta t - \left(\frac{1}{6}u_0\right)\Delta t^3 + \left(\frac{1}{4}u_0p_0\right)\Delta t^4 + O(\Delta t^5) \end{cases}$$

$$u_0 = 1/r_0^3, \quad p_0 = \vec{r}_0 \dot{\vec{r}}_0 / r_0^2, \quad q_0 = v_0^2 / r_0^2$$

$$\begin{cases} F = 1 - \frac{a}{r_0}(1 - \cos \Delta E) \\ G = \Delta t - \frac{1}{n}(\Delta E - \sin \Delta E) \end{cases}$$

$$\begin{cases} F = 1 + O(\Delta t^2), \\ G = \Delta t [1 + O(\Delta t^2)] \end{cases}$$

$$\begin{aligned} F = 1 + \frac{\tau^2}{2} \left[-u^3 + \left(\frac{3J_2}{2}\right)(5u_7z_0^2 - u_5) - \mu'u_3' \right] \\ + \frac{\tau^3}{6} \left[(3u_5\sigma) + \left(\frac{3J_2}{2}\right)(5(u_7 - 7u_9z_0^2)\sigma + 10u_7z_0\dot{z}_0) \right] \\ + \frac{\tau^4}{24} \left[u_5(3v_0^2 - 2u_1 - 15u_2\sigma^2) + \left(\frac{3J_2}{2}\right)(6u_8(4u_2z_0^2 - 1) - 5u_7(7u_2z_0^2 - 1)v_0^2 \right. \right. \\ \left. \left. + 10u_7\dot{z}_0^2 + 35u_9(9u_2z_0^2 - 1)\sigma^2 - 140u_9\sigma z_0\dot{z}_0) + u_3(\mu'u_3') \right] \\ + \frac{\tau^5}{120} u_7 [15\sigma(-3v_0^2 + 2u_1 + 7u_2\sigma^2)] + O(\tau^6) \end{aligned}$$

$$\begin{aligned} G = \tau + \frac{\tau^3}{6} \left[-u_3 + \left(\frac{3J_2}{2}\right)(5u_7z_0^2 - u_5) - \mu'u_3' \right] \\ + \frac{\tau^4}{24} \left[6u_5\sigma + \left(\frac{3J_2}{2}\right)(20u_7z_0\dot{z}_0 - 10u_7(7u_2z_0^2 - 1)\sigma) \right] \\ + \frac{\tau^5}{120} u_5 [9v_0^2 - 8u_1 - 45u_2\sigma^2] + O(\tau^6) \end{aligned}$$

改进拉普拉斯法 (二)

- 初轨确定和精密定轨的区别？
1. 条件与目的
 2. 从非线性系统的收敛性角度

$$\vec{r} = \vec{\rho} + \vec{R} \quad \vec{\rho} = \rho \hat{L}, \quad \hat{L} = (\lambda, \mu, \nu)^T$$

$$\vec{r}(t) = F^*(\vec{r}_0, \dot{\vec{r}}_0, \Delta t) \vec{r}_0 + G^*(\vec{r}_0, \dot{\vec{r}}_0, \Delta t) \dot{\vec{r}}_0$$

$$\hat{L} \times (F^* \vec{r}_0 + G^* \dot{\vec{r}}_0) = \hat{L} \times \vec{R}$$

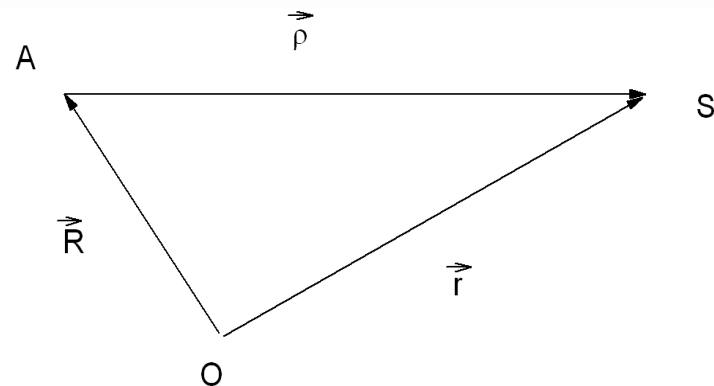
$$\begin{cases} (F\nu)x_0 - (F_z\lambda)z_0 + (G\nu)\dot{x}_0 - (G_z\lambda)\dot{z}_0 = (\nu X_e - \lambda Z_e) \\ (F\nu)y_0 - (F_z\mu)z_0 + (G\nu)\dot{y}_0 - (G_z\mu)\dot{z}_0 = (\nu Y_e - \mu Z_e) \\ (F\mu)x_0 - (F\lambda)y_0 + (G\mu)\dot{x}_0 - (G\lambda)\dot{y}_0 = (\mu X_e - \lambda Y_e) \end{cases}$$

$$\begin{cases} F^0 = 1 - \frac{1}{2r_0^3} \Delta t^2 \\ G^0 = \Delta t - \frac{\Delta t^3}{6r_0^3} \end{cases}$$

对于无任何先验信息的初轨确定，第一次迭代非常关键。从第二次迭代开始F、G可以用封闭表达式代替级数。

$$\hat{L} = \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix} = \begin{pmatrix} \cos \delta \cos \alpha \\ \cos \delta \sin \alpha \\ \sin \delta \end{pmatrix}$$

拉普拉斯方法的关键：
1. 消除距离变量
2. F/G一阶项对短弧是很好的近似



定初轨时，以上“线性系统”只是形式上是线性的，实际上是非线性的，其系数是轨道（待估）的函数。需要通过迭代完成初轨的改进。

样条基的构造

$$N_{i,2}(u) = \frac{u - u_i}{u_{i+2} - u_i} N_{i,1}(u) + \frac{u_{i+3} - u}{u_{i+3} - u_{i+1}} N_{i+1,1}(u)$$

$$= \begin{cases} \frac{(u - u_i)^2}{(u_{i+1} - u_i)(u_{i+2} - u_i)}, & u \in [u_i, u_{i+1}) \\ \frac{(u - u_i)(u_{i+2} - u)}{(u_{i+2} - u_i)(u_{i+2} - u_{i+1})} + \frac{(u - u_{i+1})(u_{i+3} - u)}{(u_{i+2} - u_{i+1})(u_{i+3} - u_{i+1})}, & u \in [u_{i+1}, u_{i+2}) \\ \frac{(u_{i+3} - u)^2}{(u_{i+3} - u_{i+1})(u_{i+3} - u_{i+2})}, & u \in [u_{i+2}, u_{i+3}) \\ 0, & \text{其他} \end{cases}$$

B样条基函数具有局部支集性

$$N'_{i,p}(u) = \frac{p}{u_{i+p} - u_i} N_{i,p-1}(u) - \frac{p}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$

$$N_{i,3}(u) = \frac{u - u_i}{u_{i+3} - u_i} N_{i,2}(u) + \frac{u_{i+4} - u}{u_{i+4} - u_{i+1}} N_{i+1,2}(u)$$

$$= \begin{cases} \frac{(u - u_i)^3}{(u_{i+1} - u_i)(u_{i+2} - u_i)(u_{i+3} - u_i)}, & u \in [u_i, u_{i+1}) \\ \frac{(u - u_i)^2(u_{i+2} - u)}{(u_{i+2} - u_i)(u_{i+2} - u_{i+1})(u_{i+3} - u_i)} + \frac{(u - u_{i+1})(u_{i+3} - u)(u - u_{i+1})}{(u_{i+2} - u_{i+1})(u_{i+3} - u_{i+1})(u_{i+2} - u_{i+1})} + \frac{(u_{i+4} - u)(u - u_{i+1})^2}{(u_{i+2} - u_{i+1})(u_{i+3} - u_{i+1})(u_{i+4} - u_{i+1})}, & u \in [u_{i+1}, u_{i+2}) \\ \frac{(u - u_i)(u_{i+3} - u)^2}{(u_{i+3} - u_i)(u_{i+3} - u_{i+1})(u_{i+3} - u_{i+2})} + \frac{(u - u_{i+1})(u_{i+3} - u)(u_{i+4} - u)}{(u_{i+3} - u_{i+1})(u_{i+3} - u_{i+2})(u_{i+4} - u_{i+1})} + \frac{(u_{i+4} - u)^2(u - u_{i+2})}{(u_{i+4} - u_{i+1})(u_{i+4} - u_{i+2})(u_{i+3} - u_{i+2})}, & u \in [u_{i+2}, u_{i+3}) \\ \frac{(u_{i+4} - u)^3}{(u_{i+4} - u_{i+1})(u_{i+4} - u_{i+2})(u_{i+4} - u_{i+3})}, & u \in [u_{i+3}, u_{i+4}) \\ 0, & \text{其他} \end{cases}$$

外测弹道确定：样条基构造

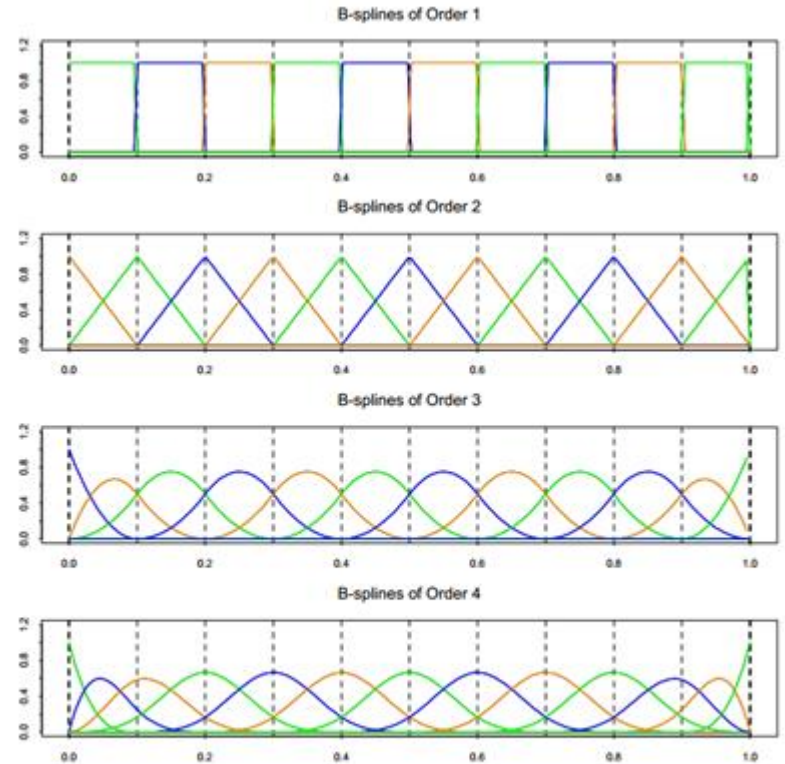
$$N_{i,0}(u) = \begin{cases} 1, & u \in [u_i, u_{i+1}) \\ 0, & \text{otherwise} \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i,p-1}(u), \quad p \geq 1$$

$$B_k = \frac{\sum_{j=0}^{k+1} (-1)^j \binom{k+1}{j} \left(x + \frac{k+1}{2} - j\right)_+^k}{k!}$$

$$u_+^0 = \begin{cases} 1, & (u > 0) \\ \frac{1}{2}, & (u = 0) \\ 0, & (u < 0) \end{cases} \quad u_+^m = \begin{cases} u^m, & (u \geq 0) \\ 0, & (u < 0) \end{cases}, \quad (m = 1, 2, \dots)$$

$N_{0,0}$	$N_{0,1}$	
$N_{1,0}$	$N_{1,1}$	$N_{0,2}$
$N_{2,0}$	$N_{2,1}$	$N_{1,2}$
$N_{3,0}$	$N_{3,1}$	$N_{2,2}$
$N_{4,0}$		$N_{3,2}$
		$N_{4,2}$
		\vdots
		\vdots
		\vdots



The sequence of B-splines up to order four with ten knots evenly spaced from 0 to 1. The B-splines have local support; they are nonzero on an interval spanned by $M + 1$ knots.

外测弹道确定：样条弹道逼近

$$\left\{ \begin{array}{l} x(t) = \sum_{j=1}^P \alpha_j B\left(\frac{t-T_j}{h}\right), \quad x(t) = \sum_{j=1}^P \frac{\alpha_j B\left(\frac{t-T_j}{h}\right)}{h} \\ y(t) = \sum_{j=1}^P \beta_j B\left(\frac{t-T_j}{h}\right), \quad y(t) = \sum_{j=1}^P \frac{\beta_j B\left(\frac{t-T_j}{h}\right)}{h}, \\ z(t) = \sum_{j=1}^P \gamma_j B\left(\frac{t-T_j}{h}\right), \quad z(t) = \sum_{j=1}^P \frac{\gamma_j B\left(\frac{t-T_j}{h}\right)}{h} \end{array} \right. \quad \left\{ \begin{array}{l} h = \frac{(T_{P-1} - T_2)}{P-3} \\ T_j = T_2 + (j-2)h \end{array} \right.$$

$$B_3 = \begin{cases} 0, (|x| \geq 2) \\ \frac{1}{2}|x|^3 - x^2 + \frac{2}{3}, (|x| \leq 1) \\ -\frac{1}{6}|x|^3 + x^2 - 2|x| + \frac{4}{3}, (1 < |x| < 2) \end{cases}$$

$$B_3(x) = \begin{cases} 0 & , |x| \geq 2 \\ \left\langle \begin{array}{l} \frac{3}{2}|x|^2 - 2x, x > 0 \\ -\frac{3}{2}|x|^2 - 2x, x < 0 \end{array} \right\rangle & , |x| < 1 \\ \left\langle \begin{array}{l} -\frac{1}{2}|x|^2 + 2x - 2, x > 0 \\ \frac{1}{2}|x|^2 + 2x + 2, x < 0 \end{array} \right\rangle & , 1 \leq |x| < 2 \end{cases}$$

外测弹道确定：线性化方程

$$O-C = \begin{bmatrix} \frac{\partial G}{\partial X} \\ \Theta \end{bmatrix} \begin{bmatrix} \Xi \\ \Theta \end{bmatrix} \Delta\psi \quad \psi = [\alpha_1, \dots, \alpha_P, \beta_1, \dots, \beta_P, \gamma_1, \dots, \gamma_P]^T$$

$$\begin{cases} B_1 = \begin{bmatrix} B\left(\frac{t_1-T_1}{h}\right) & B\left(\frac{t_1-T_2}{h}\right) & \dots & B\left(\frac{t_1-T_P}{h}\right) \\ B_2 = \begin{bmatrix} B\left(\frac{t_2-T_1}{h}\right) & B\left(\frac{t_2-T_2}{h}\right) & \dots & B\left(\frac{t_2-T_P}{h}\right) \\ \vdots \\ B_m = \begin{bmatrix} B\left(\frac{t_m-T_1}{h}\right) & B\left(\frac{t_m-T_2}{h}\right) & \dots & B\left(\frac{t_m-T_P}{h}\right) \end{bmatrix} \end{cases}$$

$$\Xi = \begin{bmatrix} B_1 & B_1 & B_1 \\ B_2 & B_2 & B_2 \\ \vdots & \vdots & \vdots \\ B_m & B_m & B_m \end{bmatrix}$$

线性问题?
初值问题?

$$\begin{cases} \dot{B}_1 = \begin{bmatrix} \dot{B}\left(\frac{t_1-T_1}{h}\right) & \dot{B}\left(\frac{t_1-T_2}{h}\right) & \dots & \dot{B}\left(\frac{t_1-T_P}{h}\right) \\ \dot{B}_2 = \begin{bmatrix} \dot{B}\left(\frac{t_2-T_1}{h}\right) & \dot{B}\left(\frac{t_2-T_2}{h}\right) & \dots & \dot{B}\left(\frac{t_2-T_P}{h}\right) \\ \vdots \\ \dot{B}_m = \begin{bmatrix} \dot{B}\left(\frac{t_m-T_1}{h}\right) & \dot{B}\left(\frac{t_m-T_2}{h}\right) & \dots & \dot{B}\left(\frac{t_m-T_P}{h}\right) \end{bmatrix} \end{cases}$$

$$\Theta = \frac{1}{h} \begin{bmatrix} \dot{B}_1 & \dot{B}_1 & \dot{B}_1 \\ \dot{B}_2 & \dot{B}_2 & \dot{B}_2 \\ \vdots & \vdots & \vdots \\ \dot{B}_m & \dot{B}_m & \dot{B}_m \end{bmatrix}$$

$$\begin{cases} X = F_1(\alpha_1, \dots, \alpha_P), \dot{X} = F_4(\alpha_1, \dots, \alpha_P) \\ Y = F_2(\beta_1, \dots, \beta_P), \dot{Y} = F_5(\beta_1, \dots, \beta_P) \\ Z = F_3(\gamma_1, \dots, \gamma_P), \dot{Z} = F_6(\gamma_1, \dots, \gamma_P) \end{cases}$$

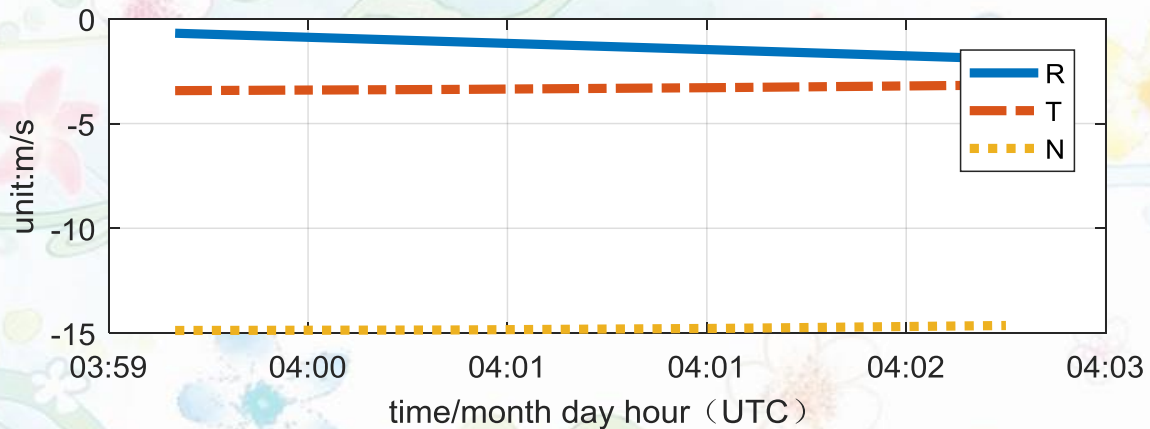
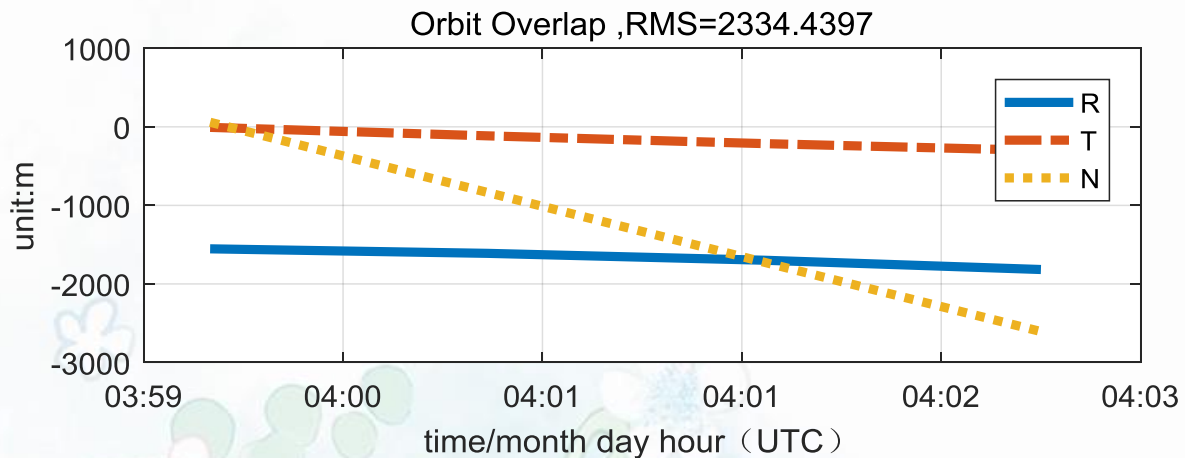
天基平台测角拉普拉斯方法 低低模式初轨确定

平台轨道	高度400km，近圆轨道，倾角42°。轨道噪声1米。
目标轨道	高度800km，近圆轨道，倾角80。
测量数据	采样率1秒，精度0.5角秒。

epoch1	2018.0	8.0	21.0
epoch2	4.0	00.0	00.000
pos 0	7278063.978	-29901.639	-12992.788
vel 0	18.289	1284.862	7288.046

2	pos 0	7298676.644	-30007.840	-13028.426
	vel 0	17.770	1114.764	7364.932
3	pos 0	7276510.626	-29953.685	-12986.493
	vel 0	17.648	1298.911	7282.058
4	pos 0	7244075.551	-29893.627	-12920.345
	vel 0	17.112	1568.733	7160.580
8	pos 0	6512857.661	-37752.246	-10076.740
	vel 0	1.227	7756.661	4390.572

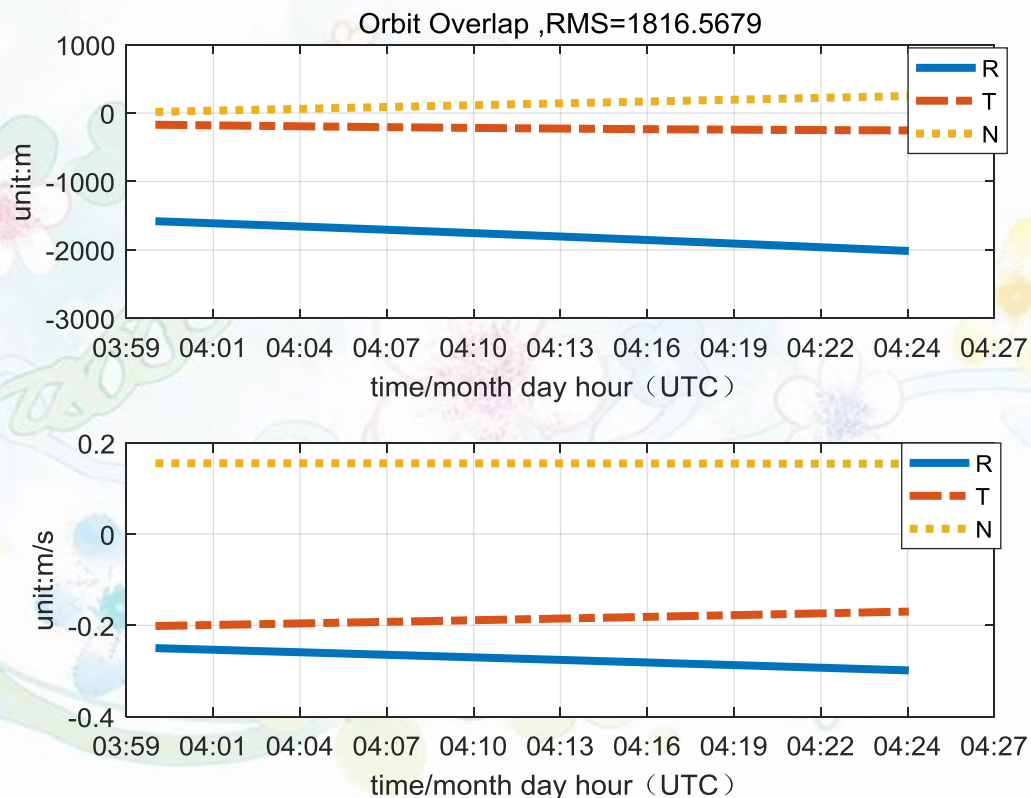
低低模式与仿真轨道比较



低高模式初轨模式

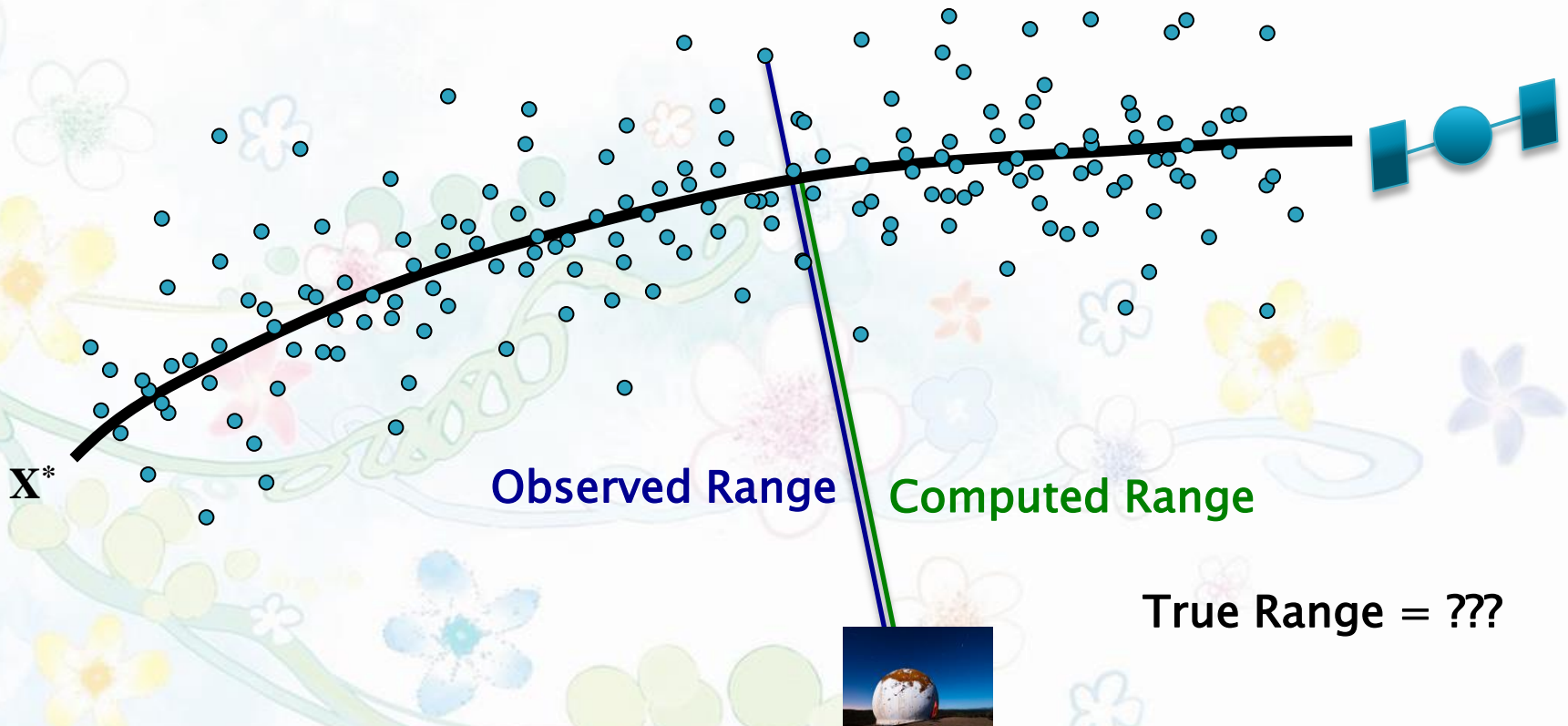
平台轨道	高度400km，近圆轨道，倾角 42° 。轨道噪声1米。
GEO轨道	星下点经度 120° 。
测量数据	采样率3秒，精度0.5角秒。

定轨弧
长为25
分钟时

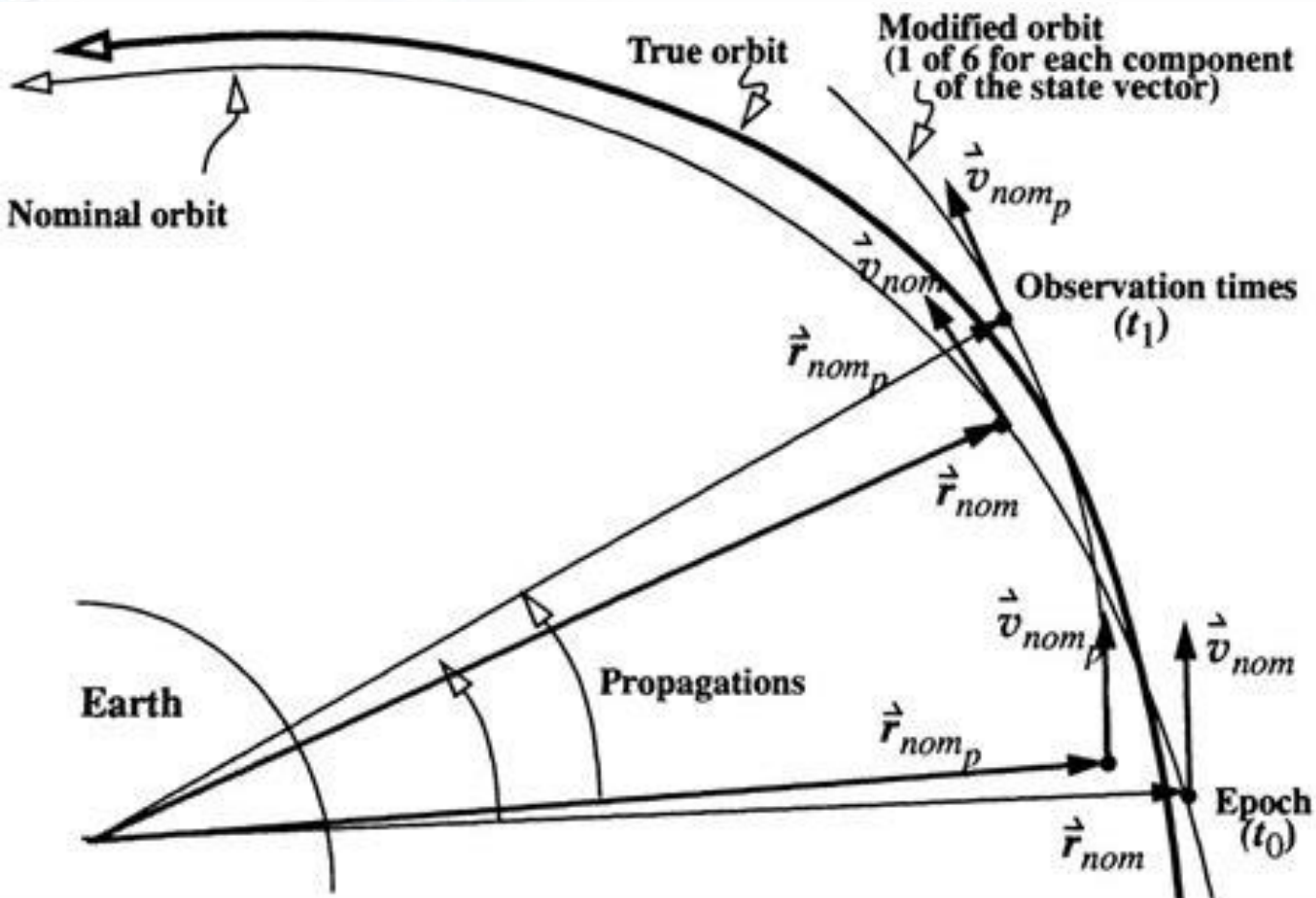
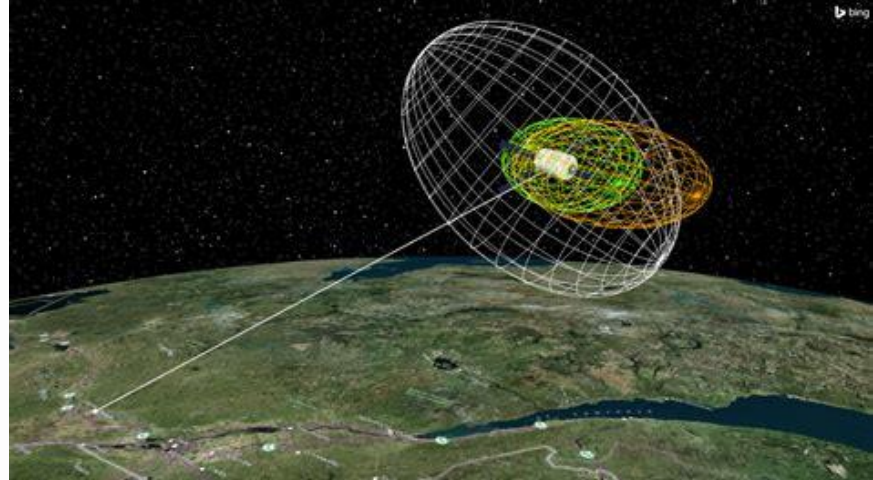


统计定轨问题描述

- ▶ 带有观测噪声的动力系统
- ▶ 需要用数学手段把各个历元的观测数据都关联到我们要估计的轨道及其他参数



多変元微分改正



一般性动力系统估计问题

$$\dot{\mathbf{X}} = F(\mathbf{X}, t) + \mathbf{u}, \quad \mathbf{X}(t_0) = \mathbf{X}_0$$

$$\mathbf{Y}_i = G(\mathbf{X}_i, t_i) + \boldsymbol{\epsilon}_i, \quad i = 1, \dots, l$$

$$\mathbf{X} \in \mathbb{R}^n, \quad \mathbf{Y}_i \in \mathbb{R}^p$$

状态向量

观测向量

观测方程和动力学方程有一个是非线性的，则该系统即为非线性系统。通常采用线性化处理。

批处理估计

$$\mathbf{y} = H \mathbf{x}_0 + \boldsymbol{\epsilon} \quad \Rightarrow \quad \mathbf{x}_0 = (H^T H)^{-1} H^T \mathbf{y}$$

$$\mathbf{y} \equiv \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_l \end{bmatrix} \quad H \equiv \begin{bmatrix} \tilde{H}_1 \Phi(t_1, t_0) \\ \tilde{H}_2 \Phi(t_2, t_0) \\ \vdots \\ \tilde{H}_l \Phi(t_l, t_0) \end{bmatrix} \quad \boldsymbol{\epsilon} \equiv \begin{bmatrix} \boldsymbol{\epsilon}_1 \\ \boldsymbol{\epsilon}_2 \\ \vdots \\ \boldsymbol{\epsilon}_l \end{bmatrix}$$

线性化

$$\mathbf{x}(t) = \mathbf{X}(t) - \mathbf{X}^*(t)$$

$$\mathbf{x}_i = \mathbf{X}(t_i) - \mathbf{X}^*(t_i)$$

$$\mathbf{y}_i = \mathbf{Y}_i - G(\mathbf{X}_i^*, t_i)$$



$$\dot{\mathbf{x}} = A(t)\mathbf{x}$$

$$\mathbf{y}_i = \tilde{H}_i \mathbf{x}_i + \epsilon_i$$

$$A(t) = \left[\frac{\partial F(\mathbf{X}, t)}{\partial \mathbf{X}(t)} \right]^*, \quad \tilde{H}_i = \left[\frac{\partial G(\mathbf{X}, t)}{\partial \mathbf{X}(t)} \right]_i^*$$

状态转移矩阵

- ▶ 线性时变微分方程

$$\dot{\boldsymbol{x}} = A(t)\boldsymbol{x}$$

- ▶ 其解的形式为

$$\boldsymbol{x}(t) = \Phi(t, t_i)\boldsymbol{x}(t_i)$$

- ▶ $\Phi(t, t_i)$ 是状态转移矩阵 (STM)，将状态 $\boldsymbol{x}(t_i)$ 映射到 $\boldsymbol{x}(t)$

状态转移矩阵的微分方程

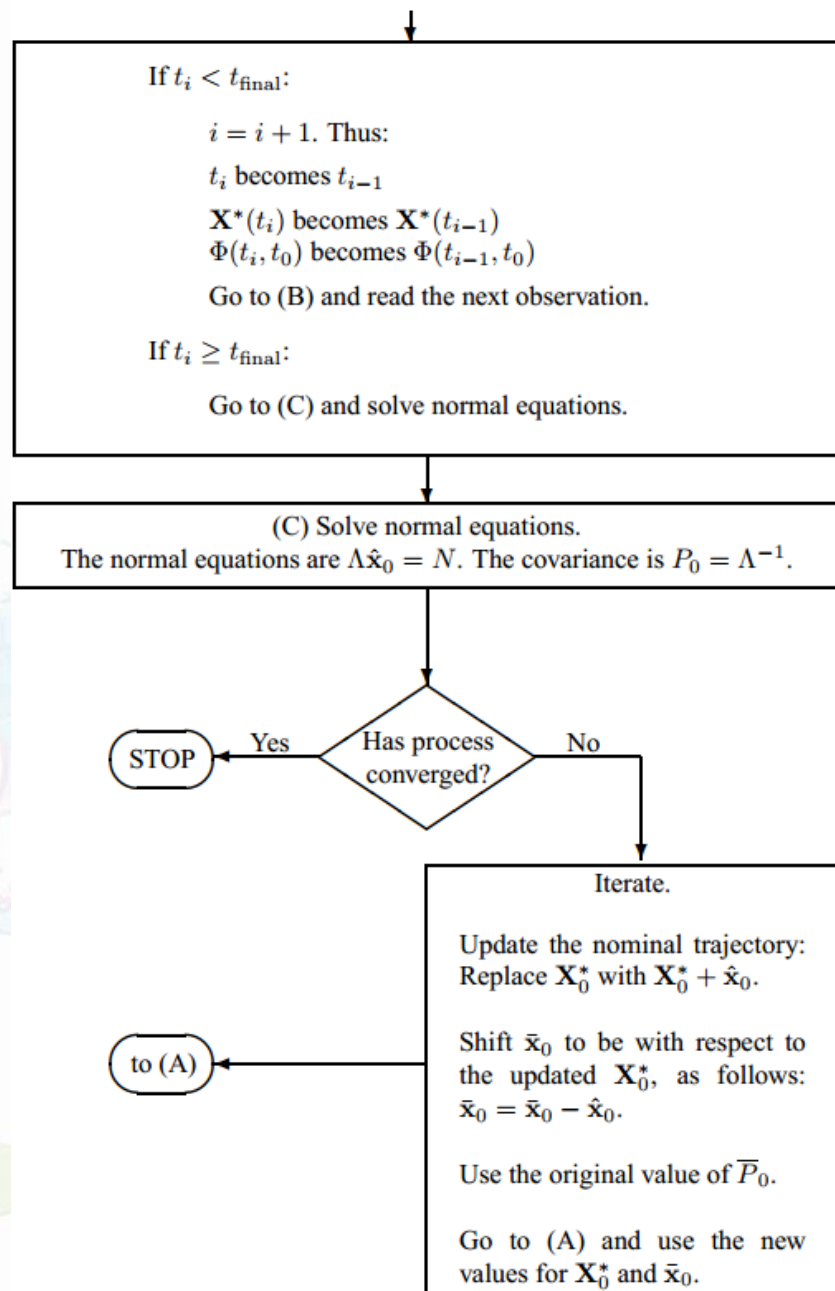
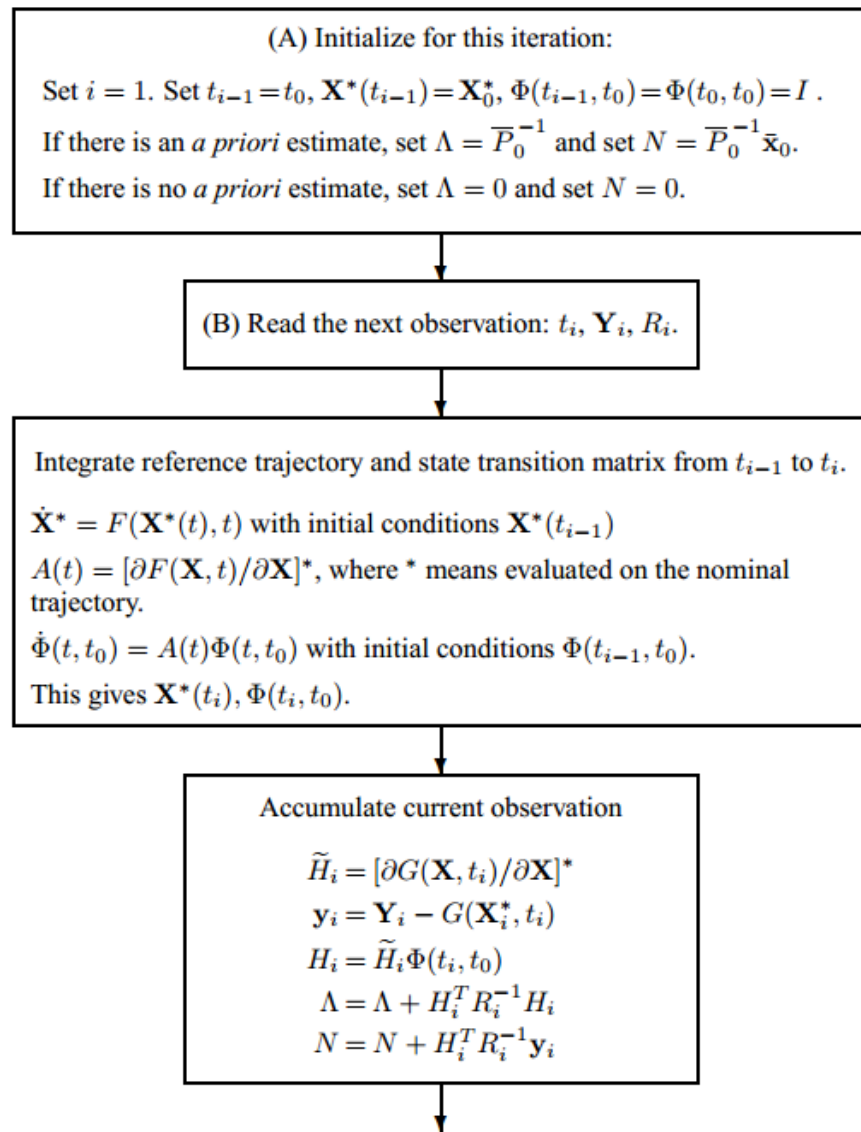
Constant!

$$\mathbf{x}(t) = \Phi(t, t_i) \mathbf{x}(t_i) \Rightarrow \dot{\mathbf{x}}(t) = \dot{\Phi}(t, t_i) \mathbf{x}(t_i)$$
$$\dot{\mathbf{x}} = A(t) \mathbf{x}$$
$$\dot{\Phi}(t, t_i) \mathbf{x}(t_i) = \dot{A}(t) \Phi(t, t_i) \mathbf{x}(t_i)$$

Why?

$$\dot{\Phi}(t, t_i) = A(t) \Phi(t, t_i), \quad \Phi(t_i, t_i) = \mathbb{I}_n$$

批处理流程





Q&A!