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参数估计方法

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2020年秋季

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线性超定方程

$$\begin{aligned}
\mathbf{y}_1 &= H_1\mathbf{x}_k + \boldsymbol{\epsilon}_1; & w_1 \\
\mathbf{y}_2 &= H_2\mathbf{x}_k + \boldsymbol{\epsilon}_2; & w_2 \\
&\vdots \quad \vdots \quad \vdots \\
\mathbf{y}_\ell &= H_\ell\mathbf{x}_k + \boldsymbol{\epsilon}_\ell; & w_\ell
\end{aligned}$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_\ell \end{bmatrix}; \quad H = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_\ell \end{bmatrix};$$

$$\boldsymbol{\epsilon} = \begin{bmatrix} \boldsymbol{\epsilon}_1 \\ \boldsymbol{\epsilon}_2 \\ \vdots \\ \boldsymbol{\epsilon}_\ell \end{bmatrix}; \quad W = \begin{bmatrix} w_1 & 0 & \cdots & 0 \\ 0 & w_2 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & w_\ell \end{bmatrix}$$

最优估值

$$J(\mathbf{x}_k) = 1/2\boldsymbol{\epsilon}^T W \boldsymbol{\epsilon} = \sum_{i=1}^{\ell} 1/2\boldsymbol{\epsilon}_i^T w_i \boldsymbol{\epsilon}_i$$

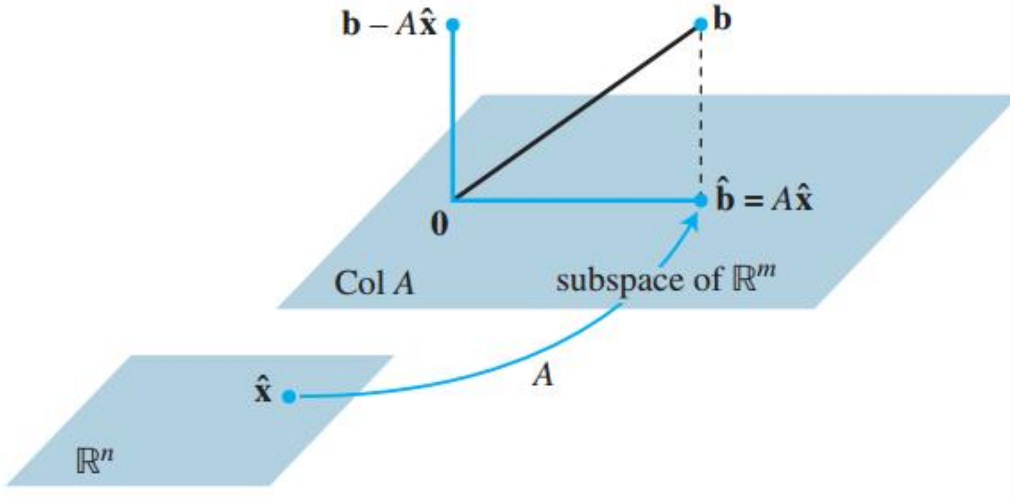
$$J(\mathbf{x}_k) = 1/2(\mathbf{y} - H\mathbf{x}_k)^T W(\mathbf{y} - H\mathbf{x}_k).$$

$$\frac{\partial J}{\partial \mathbf{x}_k} = 0 = -(y - H\mathbf{x}_k)^T W H = -H^T W(\mathbf{y} - H\mathbf{x}_k).$$

$$(H^T W H)\mathbf{x}_k = H^T W \mathbf{y}.$$

$$P_k = (H^T W H)^{-1}.$$

几何解释



The least-squares solution $\hat{\mathbf{x}}$ is in $\mathbb{R}^n$.

Suppose $\hat{\mathbf{x}}$ satisfies $A\hat{\mathbf{x}} = \hat{\mathbf{b}}$. By the Orthogonal Decomposition Theorem in Section 6.3, the projection $\hat{\mathbf{b}}$ has the property that $\mathbf{b} - \hat{\mathbf{b}}$ is orthogonal to Col A, so $\mathbf{b} - A\hat{\mathbf{x}}$ is orthogonal to each column of A. If $\mathbf{a}_j$ is any column of A, then $\mathbf{a}_j \cdot (\mathbf{b} - A\hat{\mathbf{x}}) = 0$, and $\mathbf{a}_j^T(\mathbf{b} - A\hat{\mathbf{x}}) = 0$. Since each $\mathbf{a}_j^T$ is a row of A^T,

$$
\begin{aligned}
A^T(\mathbf{b} - A\hat{\mathbf{x}}) &= \mathbf{0} \\
A^T\mathbf{b} - A^TA\hat{\mathbf{x}} &= \mathbf{0} \\
A^TA\hat{\mathbf{x}} &= A^T\mathbf{b}
\end{aligned}
$$

These calculations show that each least-squares solution of $A\mathbf{x} = \mathbf{b}$ satisfies the equation

$$A^TA\mathbf{x} = A^T\mathbf{b}$$

The matrix equation (3) represents a system of equations called the **normal equations** for $A\mathbf{x} = \mathbf{b}$. A solution of (3) is often denoted by $\hat{\mathbf{x}}$.

先验信息与序贯问题

$$\boldsymbol{y}_1 + \boldsymbol{v}_1 = \boldsymbol{A}_1\ \boldsymbol{p}_1 \qquad \text{with} \qquad \boldsymbol{D}(\boldsymbol{y}_1) = \sigma_1^2 \boldsymbol{P}_1^{-1}$$

$$\boldsymbol{y}_2 + \boldsymbol{v}_2 = \boldsymbol{A}_2\ \boldsymbol{p}_2 \qquad \text{with} \qquad \boldsymbol{D}(\boldsymbol{y}_2) = \sigma_2^2 \boldsymbol{P}_2^{-1}$$

$$\begin{bmatrix} \widehat{\boldsymbol{p}}_1 \\ \widehat{\boldsymbol{p}}_2 \end{bmatrix} + \begin{bmatrix} \boldsymbol{v}_{p_1} \\ \boldsymbol{v}_{p_2} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{I} \end{bmatrix} \widehat{\boldsymbol{p}}_c \qquad \text{with} \qquad \boldsymbol{D}\left(\begin{bmatrix} \widehat{\boldsymbol{p}}_1 \\ \widehat{\boldsymbol{p}}_2 \end{bmatrix}\right) = \sigma_c^2 \begin{bmatrix} \boldsymbol{\Sigma}_1 & \emptyset \\ \emptyset & \boldsymbol{\Sigma}_2 \end{bmatrix}.$$

$$\Big[\underbrace{\boldsymbol{A}_1^T \boldsymbol{P}_1 \boldsymbol{A}_1}_{\boldsymbol{N}_1} + \underbrace{\boldsymbol{A}_2^T \boldsymbol{P}_2 \boldsymbol{A}_2}_{\boldsymbol{N}_2}\Big] \widehat{\boldsymbol{p}}_c = \Big[\underbrace{\boldsymbol{A}_1^T \boldsymbol{P}_1 \boldsymbol{y}_1}_{\boldsymbol{b}_1} + \underbrace{\boldsymbol{A}_2^T \boldsymbol{P}_2 \boldsymbol{y}_2}_{\boldsymbol{b}_2}\Big]$$

$$\Omega_c = \sum_{i=1}^{m} \boldsymbol{y}_i^T \boldsymbol{P}_i\ \boldsymbol{y}_i - \sum_{i=1}^{m} \boldsymbol{y}_i^T \boldsymbol{P}_i \boldsymbol{A}_i \widehat{\boldsymbol{p}}_c$$

$$\widehat{\sigma}_c^2 = \frac{1}{f_c}\left(\sum_{i=1}^{m} \boldsymbol{y}_i^T \boldsymbol{P}_i\ \boldsymbol{y}_i - \sum_{i=1}^{m} \boldsymbol{y}_i^T \boldsymbol{P}_i \boldsymbol{A}_i \widehat{\boldsymbol{p}}_c\right)$$

对称正定矩阵Cholesky分解方法

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = \begin{bmatrix} l_{11} & & & \\ l_{21} & l_{22} & & \\ \cdots & & \ddots & \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & \cdots & l_{n1} \\ & l_{22} & \cdots & l_{n2} \\ & & \ddots & \\ & & & l_{nn} \end{bmatrix}$$

STEP1 $l_{11} = \sqrt{a_{11}}$

STEP2 $l_{i1} = \dfrac{a_{i1}}{l_{11}}, i = 2, N$

STEP3 对 $j = 2, N$，做 STEP4~STEP5。

STEP4 $l_{jj} = \sqrt{a_{jj} - \sum_{k=1}^{j-1} l_{jk}^2}$

STEP5 $l_{ij} = \dfrac{\left(a_{ij} - \sum_{k=1}^{j-1} l_{ik} l_{jk}\right)}{l_{jj}}, \quad i = j+1, N$

LDL分解

$$A = \begin{bmatrix} 1 & & & \\ l_{21} & 1 & & \\ \vdots & \vdots & \ddots & \\ l_{n1} & l_{n2} & \cdots & 1 \end{bmatrix} \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_n \end{bmatrix} \begin{bmatrix} 1 & l_{21} & \cdots & l_{n1} \\ & 1 & \cdots & l_{n2} \\ & & \ddots & \vdots \\ & & & 1 \end{bmatrix}$$

$$d_1 = a_{11}$$

$$\left.\begin{aligned} g_{ij} &= a_{ij} - \sum_{k=1}^{j-1} g_{ik} l_{jk} \, (j = 1, \cdots, i-1) \\ l_{ij} &= \frac{g_{ij}}{d_j} \, (j = 1, \cdots, i-1) \\ d_i &= a_{ii} - \sum_{k=1}^{i-1} g_{ik} l_{ik} \end{aligned}\right\} i = 2, \cdots, n$$

由LDL分解计算协方差

```
subroutine covariance(L,D,P,N)
!-----------------------------------------------------
! Purpose : 2019-04-15 13:08                (Created)
!   根据LDL'分解计算协方差矩阵
!-----------------------------------------------------
! Input Parameters     :
!     L    -----
!     D    ---- 对角线元素
!     N    ---- 矩阵维数
! Output Parameters    :
!-----------------------------------------------------
! Author         : Song Yezhi      <song.yz@foxmail.com>
! Copyrigt (C)   : Shanghai Astronomical Observatory,CAS
!                  (All rights reserved, 2019)
!-----------------------------------------------------
implicit none
integer     :: N
real*8      :: L(N,N),D(N),P(N,N)
!-----------------------------------------------------
integer     :: i
real*8      :: invL(N,N),invLT(N,N)
!-----------------------------------------------------
call inv_dtri(L,invL,N)
invLT = transpose(invL)
do i =1,N
    invLT(:,i)=invLT(:,i)/D(i)
end do
P = matmul(invLT,invL)
end subroutine covariance
```

快速Givens变换

$$\overbrace{\quad}^{n}\ \overbrace{\quad}^{1} \qquad\qquad \overbrace{\quad}^{n+1}$$

$$\begin{bmatrix} \overline{R} & \overline{\mathbf{b}} \\ H & \mathbf{y} \end{bmatrix} \begin{matrix} \}n \\ \}m \end{matrix} = \begin{bmatrix} \widetilde{R} \\ \widetilde{H} \end{bmatrix} \begin{matrix} \}n \\ \}m. \end{matrix}$$

Sum = 0

$U_{ii} = 1 \qquad i = 1, \ldots, n$

1. Do $k = 1, \ldots, m$

$$\delta_k = 1$$

2. Do $i = 1, \ldots, n$

If $(h_{ki} = 0)$ Go to 2

$$\begin{aligned} d'_i &= d_i + \delta_k h_{ki}^2 \\ \overline{C} &= d_i / d'_i \\ \overline{S} &= \delta_k h_{ki} / d'_i \\ y'_k &= y_k - \tilde{b}_i h_{ki} \\ \tilde{b}_i &= \tilde{b}_i \overline{C} + y_k \overline{S} \\ y_k &= y'_k \\ \delta_k &= \delta_k \overline{C} \\ d_i &= d'_i \end{aligned}$$

3. Do $j = i+1, \ldots, n$

$$\begin{aligned} h'_{kj} &= h_{kj} - U_{ij} h_{ki} \\ U_{ij} &= U_{ij} \overline{C} + h_{kj} \overline{S} \\ h_{kj} &= h'_{kj} \end{aligned}$$

Next j

Next i

$e_k = \sqrt{\delta_k} y_k$

$\text{Sum} = \text{Sum} + e_k^2$

Next k

Householder变换

如果给定向量 $\boldsymbol{x}, \boldsymbol{y}$，二者 2 范数相同，即 $\|\boldsymbol{x}\|_2 = \|\boldsymbol{y}\|_2$，则可以找到正交变换 $\boldsymbol{H}$，使 $\boldsymbol{Hx} = \boldsymbol{y}$。而变换矩阵很容易给出，即著名的 Householder 变换或称为镜像变换。取

$$\boldsymbol{u} = \frac{\boldsymbol{x} - \boldsymbol{y}}{\|\boldsymbol{x} - \boldsymbol{y}\|_2}$$

令

$$\boldsymbol{H} = \boldsymbol{I} - \boldsymbol{2uu}^T$$

即可。很容易证明 $\boldsymbol{H}$ 为正交矩阵，其变换保持向量 2 范数不变。

对于单个向量 $\boldsymbol{x}$，欲用镜像变换使之成为

$$\boldsymbol{Hx} = \boldsymbol{w} = \|\boldsymbol{x}\| \boldsymbol{e}_1, \boldsymbol{e}_1 = (1, 0, \cdots)^T$$

可以令

$$\boldsymbol{u} = \boldsymbol{w} - \boldsymbol{x}$$

继而可以构造镜像变换矩阵

$$\boldsymbol{H} = \boldsymbol{I} - 2\frac{\boldsymbol{uu}^T}{\|\boldsymbol{u}\|_2^2}$$

$\|\boldsymbol{u}\|_2^2$ 表示向量之 2 范数的平方。

Householder变换

实施变换时为了让作为分母的 $\|\boldsymbol{u}\|_2^2$ 尽可能的大，从而有利于数值稳定，取

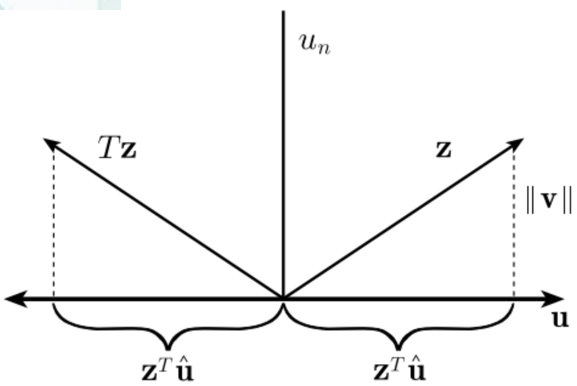
$$k = -\mathrm{sgn}\left(x_1\right)\|\boldsymbol{x}\|_2, \mathrm{sgn}\left(x_1\right) = \begin{cases} 1, x_1 \geqslant 0 \\ -1, x_1 < 0 \end{cases}$$

$$\boldsymbol{u} = \left(x_1 + \mathrm{sgn}\left(x_1\right)\|\boldsymbol{x}\|_2, x_2, \cdots, x_m\right)^T$$

意义很明了，及当 $\boldsymbol{x}$ 的第一个分量大于等于 0 时候，$\boldsymbol{u}$ 的第一个分量取 x_1 与 $\|\boldsymbol{x}\|_2$ 的和，当 $\boldsymbol{x}$ 的第一个分量小于 0 时候，$\boldsymbol{u}$ 的第一个分量取 $x_1 - \|\boldsymbol{x}\|_2$。如果不这样做有可能会损失有效位数。

对于矩阵 $\boldsymbol{A}$ 做 QR 分解，即连续使用 Houlseholder 变换。如第一次使用正交变换后，使之成为如下形式：

$$\boldsymbol{H}_1\boldsymbol{A} = \begin{bmatrix} * & * & * & \cdots & * \\ 0 & * & * & \cdots & * \\ 0 & * & * & \cdots & * \\ 0 & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & * & * & \cdots & * \end{bmatrix}$$



Householder变换

第二次变换使之成为如下形式：

$$
\boldsymbol{H}_2\boldsymbol{H}_1\boldsymbol{A} = \begin{bmatrix} * & * & * & \cdots & * \\ 0 & * & * & \cdots & * \\ 0 & 0 & * & \cdots & * \\ 0 & 0 & * & \cdots & * \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & * & \cdots & * \end{bmatrix}
$$

如此循环到矩阵的最后一列，至此变换后的矩阵已经成为上三角矩阵。而变换矩阵即 $\boldsymbol{H} = \cdots \boldsymbol{H}_2\boldsymbol{H}_1$。

由数值代数理论可知，如果 $\boldsymbol{R}$ 对角元素正负号都选正号或者负号，则 QR 分解是唯一的。

计算 $\boldsymbol{H}$ 时候，并不需要存储各次的变换结果，而是逐步矩阵累成而得，变换到矩阵的最后一列，新的矩阵已经是 $\boldsymbol{H}$。上三角阵也不需要再执行 $\boldsymbol{R} = \boldsymbol{H}^T\boldsymbol{A}$，当变换到最后一列时已经自动形成上三角阵，该矩阵即为 $\boldsymbol{R}$。

修正的Gram-Schimdt 正交化方法

设 $\{\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_n\}$ 是 p 维向量子空间 W 的任意一组基，则子空间 W 的标准正交基 $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_n\}$ 可以通过 Gram-Schimdt 正交化构造，这个方法是大家都很熟悉的，即

$$\boldsymbol{p}_1 = \boldsymbol{x}_1, \boldsymbol{u}_1 = \frac{\boldsymbol{p}_1}{\|\boldsymbol{p}\|} = \frac{\boldsymbol{x}_1}{\|\boldsymbol{x}_1\|}$$

$$\boldsymbol{p}_k = \boldsymbol{x}_k - \sum_{i=1}^{k=1} \left(\boldsymbol{v}_i^H \boldsymbol{x}_k\right) \boldsymbol{u}_i, \boldsymbol{u}_k = \frac{\boldsymbol{p}_k}{\|\boldsymbol{p}_k\|}$$

对于超定的线性方程系数矩阵的 QR 分解，可以通过 Gram-Schimdt 正交化方法来实现，然而采用 Gram-Schimdt 正交化方法求解列正交矩阵 $\boldsymbol{Q}$ 时，舍入误差较大，这在求解最小二乘法时候，有时会不稳定。针对 Gram-Schimdt 正交化的缺点，下面给出修正的 Gram-Schimdt 正交化算法。

修正的Gram-Schimdt 正交化方法

对于 n 个向量 $\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n$ 构造标准正交基 $\boldsymbol{q}_1, \boldsymbol{q}_2, \cdots, \boldsymbol{q}_n$ 方法如下：

$$R_{11} = \|\boldsymbol{a}_1\|$$

$$\boldsymbol{q}_1 = \frac{\boldsymbol{q}_1}{R_{11}}$$

对于 $k = 2, \cdots, n$

$$R_{jk} = \boldsymbol{q}_j^H \boldsymbol{a}_k, j = 1, \cdots, k-1$$

$$R_{kk} = \left\| \boldsymbol{a}_k - \sum_{j=1}^{k-1} \boldsymbol{q} R_{jk} \right\|$$

$$\boldsymbol{q} = \frac{\boldsymbol{a}_k - \sum_{j=1}^{k-1} \boldsymbol{q} R_{jk}}{R_{kk}}$$

扩展卡尔曼滤波

Given: P_{k-1}, $\hat{\mathbf{X}}_{k-1}$ and $\mathbf{Y}_k$, R_k.

(1) Integrate from t_{k-1} to t_k,

$$\begin{aligned} \dot{\mathbf{X}}^* &= F(\mathbf{X}^*, t), & \mathbf{X}^*(t_{k-1}) &= \hat{\mathbf{X}}_{k-1} \\ \dot{\Phi}(t, t_{k-1}) &= A(t)\Phi(t, t_{k-1}), & \Phi(t_{k-1}, t_{k-1}) &= I. \end{aligned}$$

(2) Compute

$$\begin{aligned} \overline{P}_k &= \Phi(t_k, t_{k-1})P_{k-1}\Phi^T(t_k, t_{k-1}) \\ \mathbf{y}_k &= \mathbf{Y}_k - G(\mathbf{X}_k^*, t_k) \\ \widetilde{H}_k &= \partial G(\mathbf{X}_k^*, t_k)/\partial \mathbf{X}_k. \end{aligned}$$

(3) Compute

$$\begin{aligned} K_k &= \overline{P}_k \widetilde{H}_k^T [\widetilde{H}_k \overline{P}_k \widetilde{H}_k^T + R_k]^{-1} \\ \hat{\mathbf{X}}_k &= \mathbf{X}_k^* + K_k \mathbf{y}_k \\ P_k &= [I - K_k \widetilde{H}_k]\overline{P}_k. \end{aligned}$$

(4) Replace k with $k+1$ and return to (1).

cmatlib矩阵类库定义（C++）

```
class VEC
/*-------------------------------------------------------
   Versions and Changes :
      v1.0----2015-7-18
          vector class
          下标索引算符为()
---------------------------------------------------------
   Public Paras:
          int size1 ---------  rows  of matrix
          int size2 ---------  cols  of matri
   Funcitons :
          VEC (M)  ---- creat a vector with lenght of M
          operator() --- access the component
---------------------------------------------------------
   Author       : 宋叶志  <song.yz@foxmail.com>
   Copyrigt(C)  : Shanghai Astronomical Observatory, CAS
                  (All rights reserved)           2015
---------------------------------------------------------*/
{
   private:
      double *x ;
      int M1;
   public:
      // Constructors
      VEC (int M)
      //creat a vector
      {
        M1=M;
        x= new double [M] ;
        for(int i=0;i<M;i++)
          x[i]=0.0;
      }
      // Destructor
      ~ VEC ()
      { delete [] x; }

      double& operator() (int i)
      // Component access
      { return x[i]; }
      double  operator () (int i) const { return x[i]; };
      int size() const { return M1; };
      void  output(int width,int precision);
      //output to the screen
      void  setv(double value);
      //set the elements of the vector to a double value
     //+,-,*,/ for the same index of two vectors
     //which means that the size of the vectors must be the same
     friend VEC operator + (const VEC& V1, const VEC& V2); //V1+V2
     friend VEC operator - (const VEC& V1, const VEC& V2); //V1-V2
     friend VEC operator * (const VEC& V1, const VEC& V2); //V1.*V2,
     friend VEC operator / (const VEC& V1, const VEC& V2); //V1./V2,
     //V1./V2
     //+,-,*,/ for the vector and a scala
     // the operation works on every elements of the vector
     friend VEC operator + (const VEC& V1, double a);     //V1+a
     friend VEC operator - (const VEC& V1, double a);     //V1-a
     friend VEC operator * (const VEC& V1, double a);     //V1*a
     friend VEC operator / (const VEC& V1, double a);     //V1/a
} ;
```

cmatlib矩阵类库定义

```
class MAT
/*--------------------------------------------------------
   Versions and Changes :
      v1.0----2015-7-18
         matrix class
          在数组基础上构造,下标算符重载()
--------------------------------------------------------
   Public Paras:
           int size1 ---------  rows  of matrix
           int size2 ---------  cols  of matrix
   Functions   :
           MAT(M,N)  ---- creat a vector with the size of (M,N)
           operator() ---  caccess the component
--------------------------------------------------------
   Author      : 宋叶志  <song.yz@foxmail.com>
   Copyrigt(C) : Shanghai Astronomical Observatory, CAS
                 (All rights reserved)          2015
-------------------------------------------------------*/
{
   private:
     double ** A;
     int M1,N1;
   public:
     // Constructors
     MAT(int M,int N)
     {
        M1=M;
        N1=N;

        int i,j;

        A= new double *[M];
        for(i=0;i<M;i++)
          A[i]=new double [N] ;


        // set the initial value to zero
        for (i=0;i<M;i++)
         for(j=0;j<N;j++)
           A[i][j]=0.0;
     }
     // Destructor
     ~MAT()
     {
        int i;
        for(i=0;i<M1;i++)
          delete[] A[i];
        delete[] A;
     }

    //Component access
     double   operator() (int i,int j) const  { return A[i][j]; };
     double & operator() (int i,int j)        { return A[i][j]; };


     int size1() const { return M1; };
     int size2() const { return N1; };

     void output(int width,int precision);
     //output to the screen
     void setv(double value);
     //set the elements of the matrix to a double value

     //+,-,*,/ for the same index of two matrixs
     //which means that the size of the matrixs must be the same
     friend MAT operator + (const MAT& A1, const MAT& A2);   //A1+A2
     friend MAT operator - (const MAT& A1, const MAT& A2);   //A1-A2
     friend MAT operator * (const MAT& A1, const MAT& A2);   //A1.*A2
     friend MAT operator / (const MAT& A1, const MAT& A2);   //A1./A2

     //+,-,*,/ for the matrix and a scala
     // the operation works on every elements of the matrix
     friend MAT operator + (const MAT& A1, double a);      //A1+a
     friend MAT operator - (const MAT& A1, double a);      //A1-a
     friend MAT operator * (const MAT& A1, double a);      //A1*a
     friend MAT operator / (const MAT& A1, double a);      //A1/a

};
```

cmatlib向量运算与矩阵分解

```
//---------------基本向量运算
void veccopy(VEC &b,const VEC &a);
//按值复制一个向量
double vecdot(const VEC &v1,const VEC & v2);
//向量内积
double norm(const VEC &v);
//向量2范数
//-----------------基本矩阵运算
void matcopy(MAT &B,const MAT &A);
//矩阵复制
void transpose(MAT &AT,const MAT &A);
//矩阵转置
void matmul(VEC &b,const MAT &A,const VEC &x);
// 矩阵乘以向量
void matmul(MAT &C,const MAT &A,const MAT &B);
//矩阵乘以矩阵
```

```
//-----------------矩阵分解与求逆
void LDL(const MAT &A,MAT &L,VEC &D);
// LDL 分解
void MGS(const MAT &A, MAT &Q, MAT &R);
//修正的Gram-Schmidt正交化方法
void householder(const MAT &A,MAT &Q,MAT &R);
//hoseholder正交变换
void invlowtri(const MAT &R,MAT &S);
//inverse of lower triangula matrix
void invuptri(const MAT &U,MAT &R);
//inverse of upper triangula matrix
void invmat(const MAT &A,MAT &invA);
//对称正定矩阵逆矩阵
void inv(const MAT &A,MAT &iA);
//一般矩阵的逆矩阵 采用MGS分解方法
```

cmatlib求解线性代数方程

```
//---------------------线性代数方程
void LS_LDL(VEC &x,const MAT &A,const VEC &b);
//基于不开平方的cholesky分解计算最小二乘问题， A为对称正定矩阵
//void LS_hous(VEC &x,const MAT &A,const VEC &b);
//基于Householder变换求解最新二乘问题或适定问题
void LS_MGS( VEC &x,const MAT &A,const VEC &b);
//通过修正的Gram-Schimdt正交化求解最小二乘问题或适定问题
void LS_hous( VEC &x,const MAT &A,const VEC &b);
//least square solution or general linear equation
void uptri(const MAT &A,const VEC &b,VEC &x);
//上三角矩阵方程计算
void lowtri(const MAT &A, const VEC &b, VEC &x);
//下三角矩阵方程计算
void downtri(const MAT &A, const VEC &b, VEC &x);
//-------------------插值算法
void lagrange(const VEC &x,const VEC &y,const VEC &xx,VEC &yy);
//lagrange interp
void rot_mat(MAT &Rmat,double angle,char axID);
//rotation matrix
double robustweight(double v,double sigma);
```

下载地址：http://202.127.29.4/astrodynamics/course.php


Q&A!