中国科学院上海天文台

中国科学院大学
University of Chinese Academy of Sciences

空间飞行器精密定轨

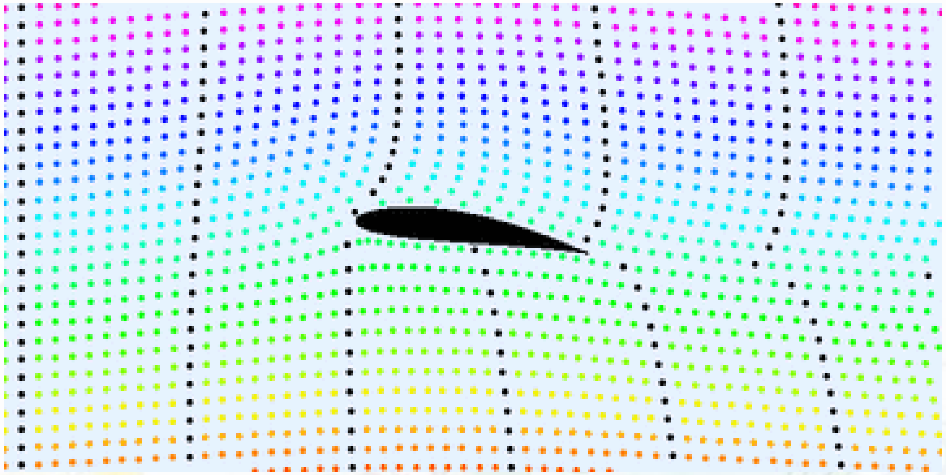
宋叶志

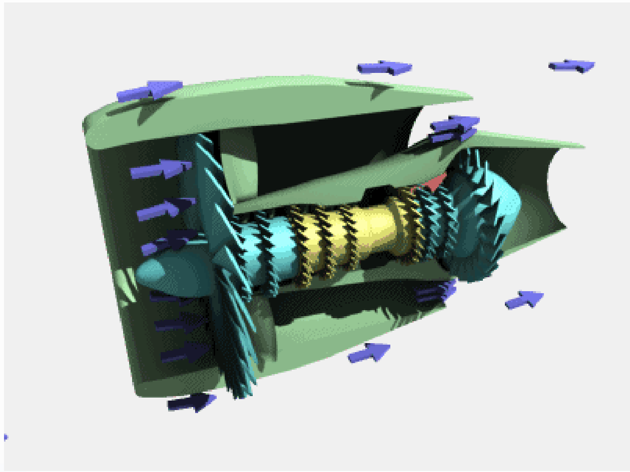
2021秋季　　作业邮箱：song.yz@foxmail.com
课件地址：http://202.127.29.4/astrodynamics

第八讲 一般摄动法

- 卫星受力情况
- 摄动方程的建立
- 一般摄动法简介
- 半解析法
- 摄动法应用

航空器与航天器飞行原理的区别



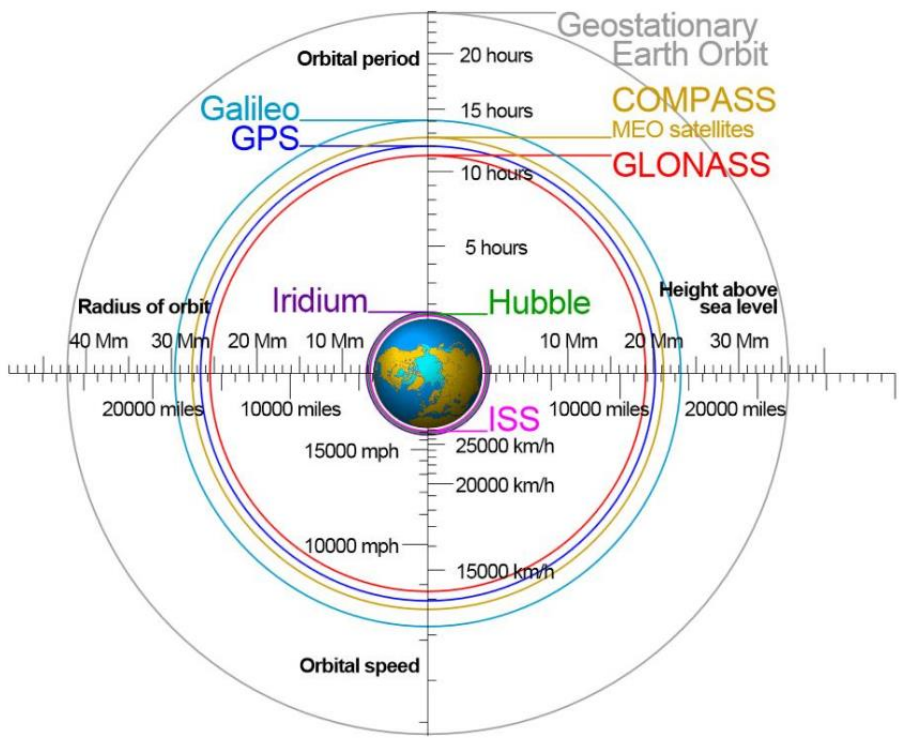


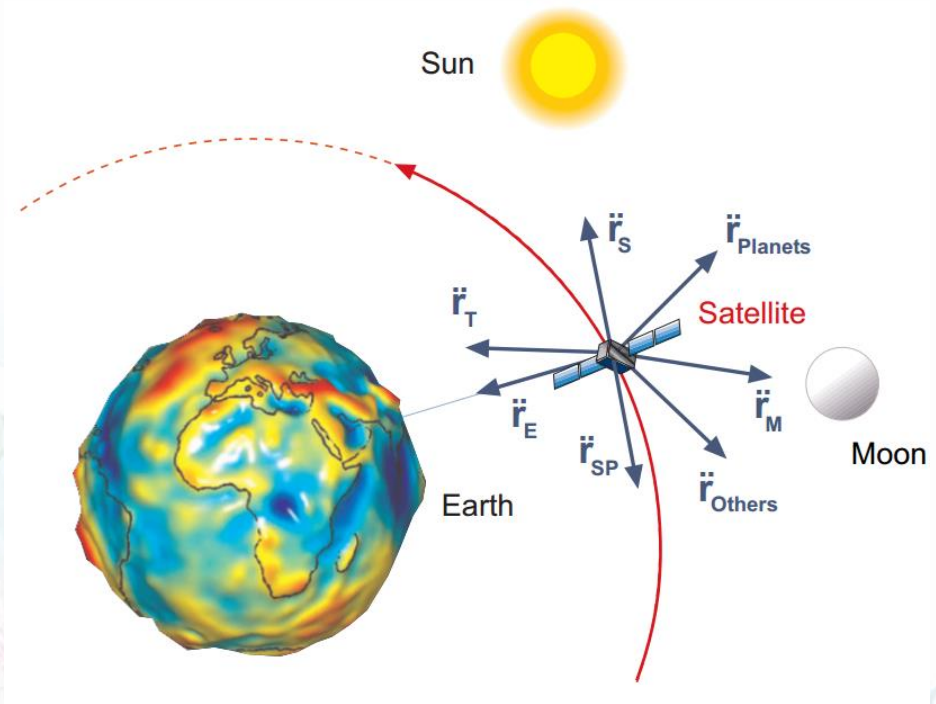
航天器飞行原理则满足基本的天体力学规律

伯努利原理：动能+重力势能+压力势能=常数

飞机飞行时机翼周围空气的流线分布是指机翼横截面的形状上下不对称,机翼上方的流线密,流速大,下方的流线疏,流速小。由伯努利方程可知,机翼上方的压强小,下方的压强大。这样就产生了作用在机翼上的方向的升力。

真实的卫星运动

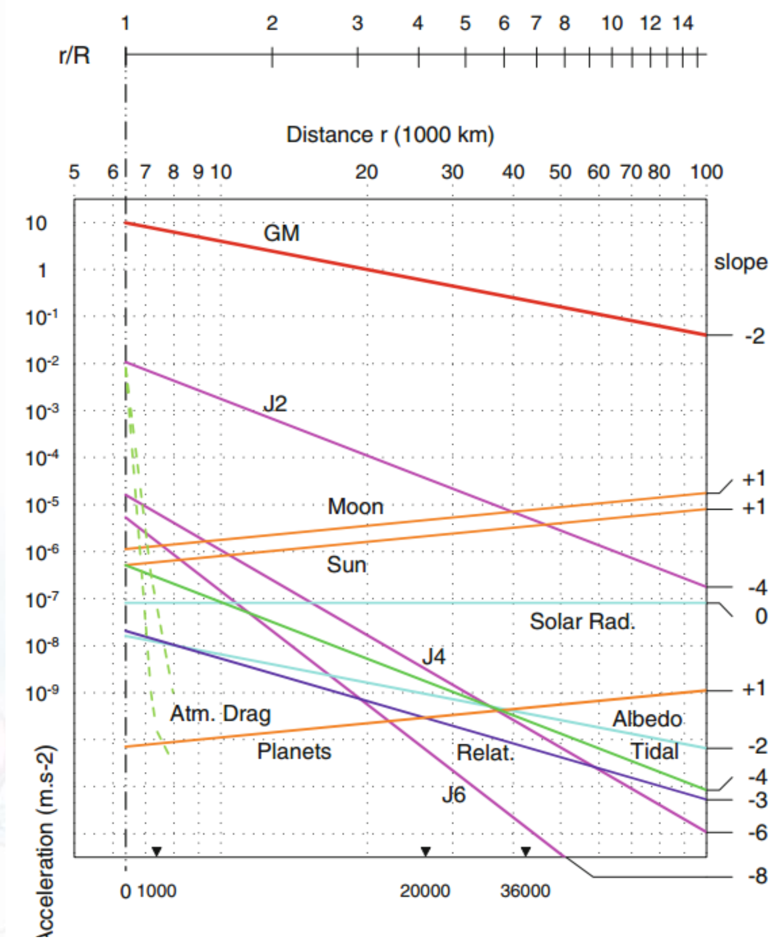




- $h = 1{,}000\,\text{km}$ for satellites in low orbit (LEO),
- $h = 20{,}000\,\text{km}$ for positioning satellites (MEO),
- $h = 36{,}000\,\text{km}$ for geostationary satellites (GEO).

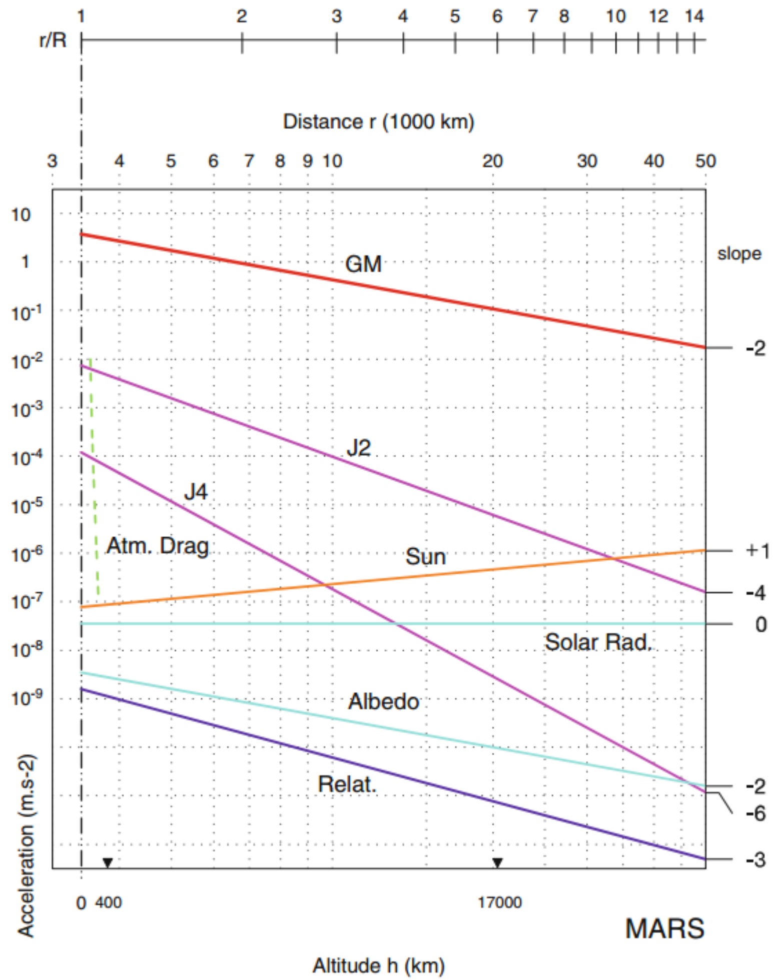
地球卫星摄动力量级

Symbol		Type of force
C		Conservative forces
	CC	• Attraction of the Earth
	CCC	• ∘ Central term $\mu = GM$
	CCN	• ∘ Other terms than CCC
	CL	• Attraction of the Moon
	CS	• Attraction of the Sun
	CP	• Attraction by other planets
	CT	• Tidal effects (land, oceans)
	CR	• Relativistic effects
D		Dissipative forces
	DF	• Atmospheric drag
	DP	• Solar radiation pressure
	DA	• Albedo effect



- 10^{-3} for the perturbation due to the flattening of the Earth,
- 10^{-6} for perturbations due to other irregularities of the geoid,
- 10^{-7} for the attraction of the Moon,
- 10^{-8} for the attraction of the Sun.

火星卫星摄动量级

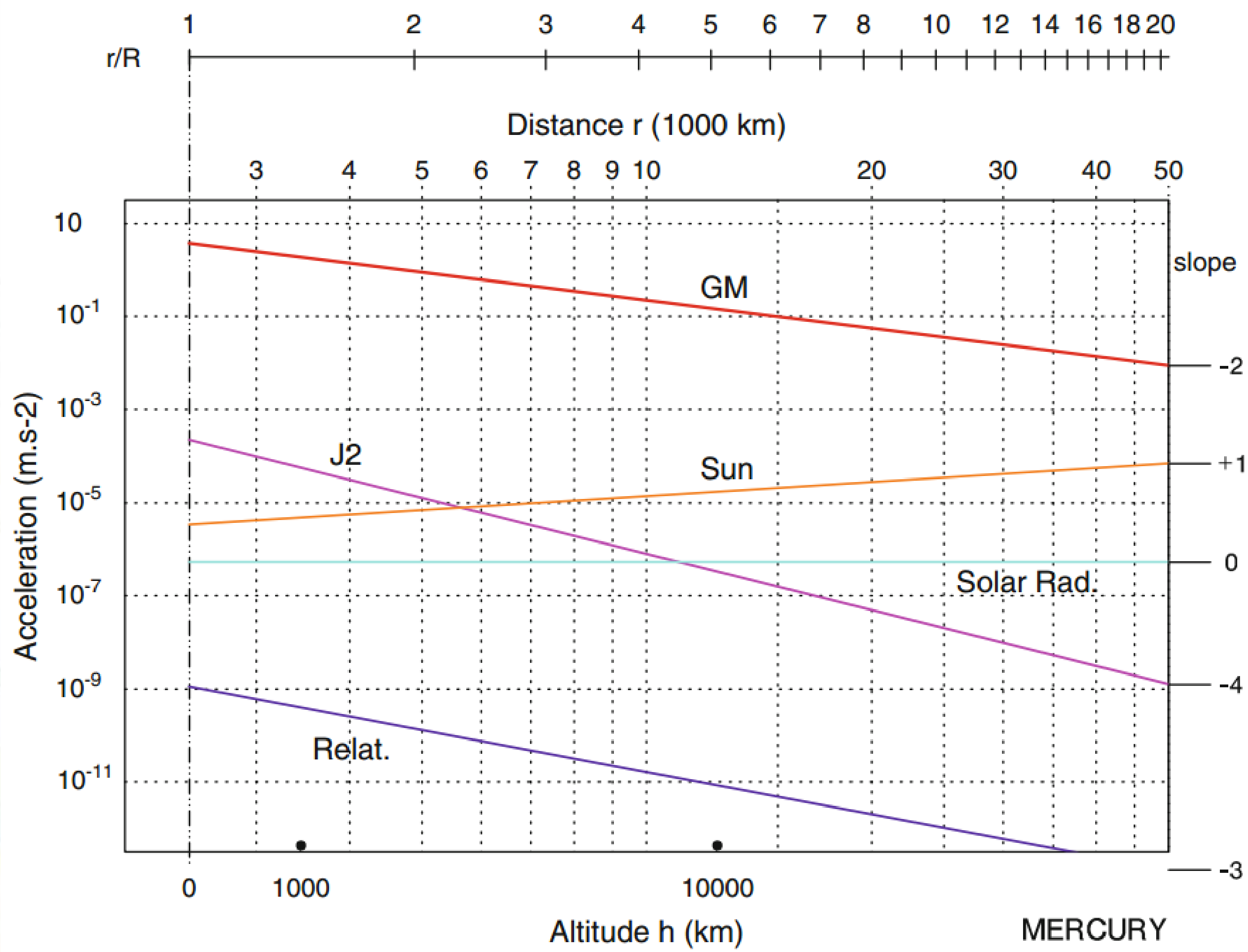


Central and perturbative accelerations as a function of the distance r of the satellite from the center of Mars. Log–log scale. In the range of variation considered, the curves are approximately straight lines and their slope is indicated. The altitudes of two types of satellite have also been noted.

Quantity	Symbol	Unit	Mars	Earth
Gravitational constant	$\mu = GM$	$m^3 s^{-2}$	4.2828369 10^{13}	3.9860044 10^{14}
Equatorial radius	R	km	3,397.000	6,378.137
Flattening	$1/f$	–	154.40915	298.25766
Equatorial circumference	$L_{P/equat}$	km	21,343.980	40,075.012
Meridian circumference	L_M	km	21,274.922	40,007.860
Acceleration (equatorial)	g	$m\,s^{-2}$	3.7052	9.7803
Acceleration (45°)	g	$m\,s^{-2}$	3.7214	9.8061
Acceleration (pole)	g	ms^{-2}	3.7376	9.8321
Gravitational potential	$J_2 \times 10^6$	–	1,955.4513	1,082.6267
Gravitational potential	$J_3 \times 10^6$	–	+31.4559	−2.5327
Gravitational potential	$J_4 \times 10^6$	–	−15.3694	−1.6196
Ratio (frozen orbit)	$J_3/J_2 \times 10^3$	–	+16.0863	−2.3394
Semi-major axis	a	a.u.	1.52366	1.00000
Period of revolution				
sidereal	N_{sid}	day	686.9800	365.2564
tropical	N_{tro}	day	686.9725	365.2422
anomalistic	N_{ano}	day	686.9951	365.2496
Angular speed	$\dot{\Omega}_T \times 10^5$	$rad\,s^{-1}$	7.088218	7.292115
	$\dot{\Omega}_T$	$°d^{-1}$	350.891983	360.985559
Period of rotation				
sidereal	D_{sid}	h	24.622962	23.934471
		s	88,642.663	86,164.090
mean solar day	D_M	h	24.6598	24.0000
		s	88,775.245	86,400.000
Inclination/ecliptic	i	deg	1.8496	–
Obliquity	ε	deg	25.19	23.44
Eccentricity	e	–	0.0930	0.01671

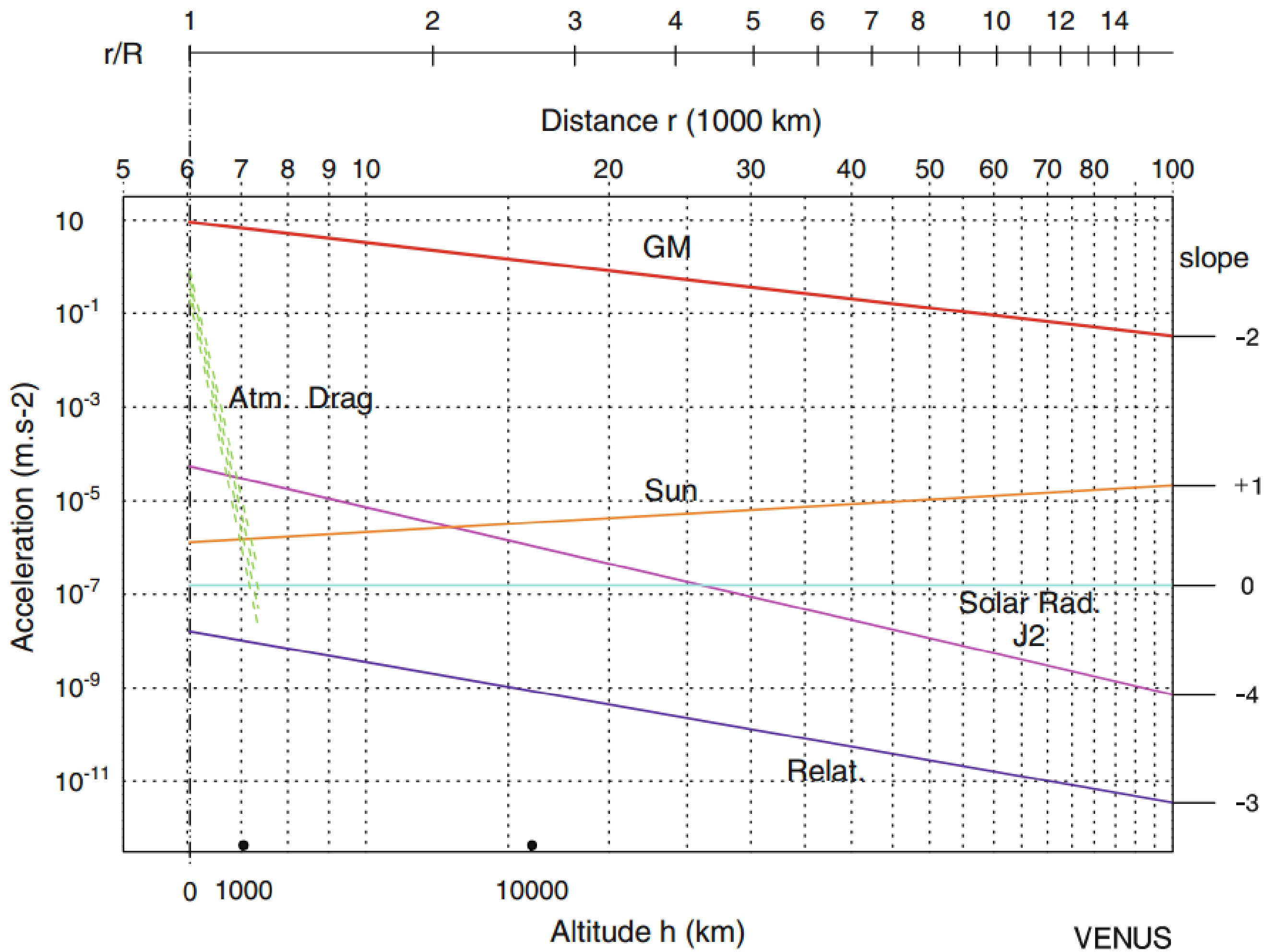
Geodetic and astronomical data for Mars and the Earth. For the units, a dash means dimensionless.

水星卫星摄动量级



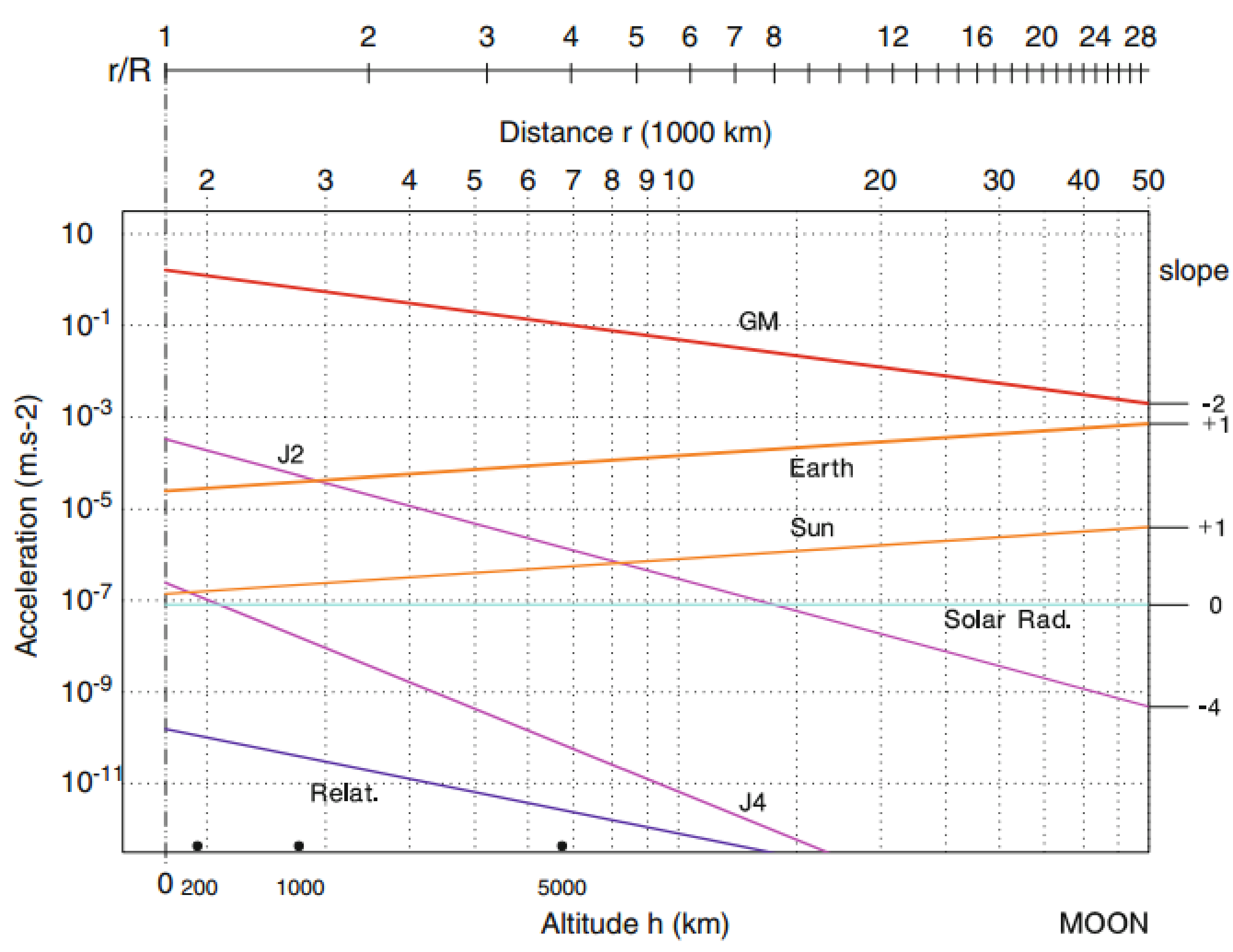
r/R
1
2
3
4
5
6
7
8
10
12
14
16
18
20
Distance r (1000 km)
3
4
5
6
7
8
9
10
20
30
40
50
10
10-1
10-3
10-5
10-7
10-9
10-11
Acceleration (m.s-2)
GM
J2
Sun
Solar Rad.
Relat.
slope
-2
+1
0
-4
-3
0
1000
10000
Altitude h (km)
MERCURY

金星卫星摄动量级



r/R
1
2
3
4
5
6
7
8
10
12
14
Distance r (1000 km)
5
6
7
8
9
10
20
30
40
50
60
70
80
100
10
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
Acceleration (m.s-2)
GM
slope
-2
Atm. Drag
Sun
+1
0
Solar Rad.
J2
-4
Relat.
-3
0
1000
10000
Altitude h (km)
VENUS

月球卫星摄动量级



r/R
1 2 3 4 5 6 7 8 12 16 20 24 28
Distance r (1000 km)
2 3 4 5 6 7 8 9 10 20 30 40 50
Acceleration (m.s-2)
10
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
slope
GM
J2
Earth
Sun
Solar Rad.
Relat.
J4
-2
+1
+1
0
-4
0 200 1000 5000
Altitude h (km)
MOON

常数变易法

无摄积分常数

$$\mathbf{c}=\begin{pmatrix}a & e & \omega & \Omega & i & M\end{pmatrix}^T \equiv \begin{pmatrix}c_1 & c_2 & c_3 & c_4 & c_5 & c_6\end{pmatrix}^T$$

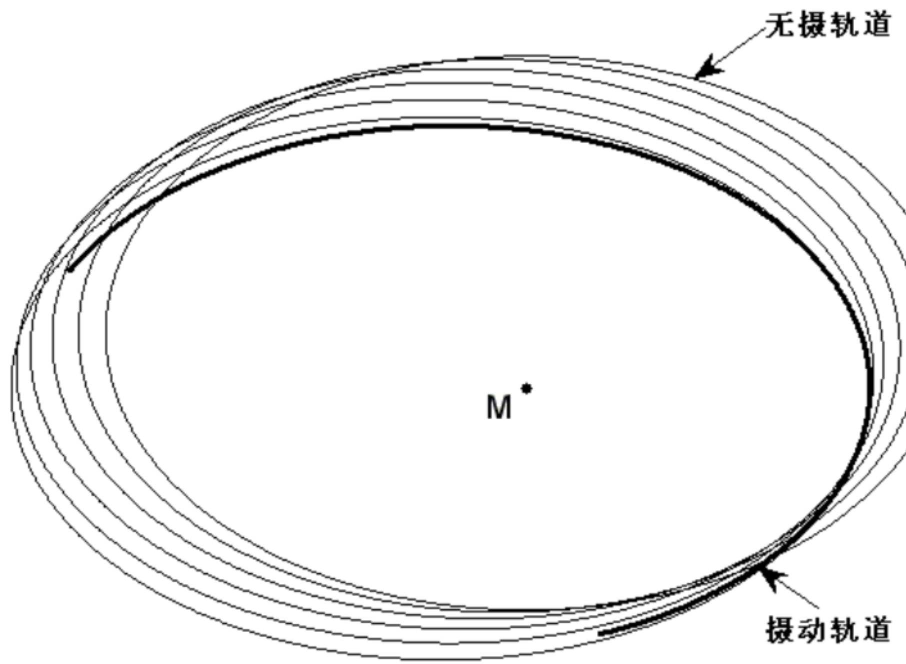
状态量可以写为积分常数和时间的函数

$$\mathbf{r}=\mathbf{r}(\mathbf{c},t), \quad \mathbf{v}=\dot{\mathbf{r}}(\mathbf{c},t)$$

摄动轨道在每一瞬间都与一条无摄轨道相切，这些无摄轨道组成一个族，摄动轨道则是无摄轨道族的包络 。

常数变易法

根数c 也不再是常量，它在每一瞬时的值确定此时与摄动轨道相吻切的无摄轨道 。



摄动轨道是无摄轨道族的包络

常数变易法

由于根数c 不再是常量，速度向量是向径对时间的全导数

$$\dot{\mathbf{r}}(\mathbf{c},t)=\frac{\partial\mathbf{r}(\mathbf{c},t)}{\partial t}+\frac{d\mathbf{r}(\mathbf{c},t)}{d\mathbf{c}^T}\dot{\mathbf{c}}.$$

$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{pmatrix}=\begin{pmatrix}\frac{\partial x}{\partial t}\\ \frac{\partial y}{\partial t}\\ \frac{\partial z}{\partial t}\end{pmatrix}+\begin{pmatrix}\frac{\partial x}{\partial c_1} & \frac{\partial x}{\partial c_2} & \cdots & \frac{\partial x}{\partial c_6}\\ \frac{\partial y}{\partial c_1} & \frac{\partial y}{\partial c_2} & \cdots & \frac{\partial y}{\partial c_6}\\ \frac{\partial z}{\partial c_1} & \frac{\partial z}{\partial c_2} & \cdots & \frac{\partial z}{\partial c_6}\end{pmatrix}\begin{pmatrix}\dot{c}_1\\ \dot{c}_2\\ \vdots\\ \dot{c}_6\end{pmatrix}.$$

等于吻切无摄轨道的速度向量，这时要把c 看做常量

$$\dot{\mathbf{r}}(\mathbf{c},t)=\frac{\partial\mathbf{r}(\mathbf{c},t)}{\partial t}$$

第二项应该为零

$$\frac{d\mathbf{r}(\mathbf{c},t)}{d\mathbf{c}^T}\dot{\mathbf{c}}=\mathbf{0}$$

常数变易法

摄动加速度表示为摄动势函数的梯度，
则有摄运动方程为

$$\ddot{\mathbf{r}}(\mathbf{c},t)+\frac{\mu}{r^3}\mathbf{r}(\mathbf{c},t)=\frac{dR}{d\mathbf{r}}.$$

当摄动加速度为零时， c 成为常量，方程退化为无摄二体运动方程

$$\frac{\partial^2\mathbf{r}}{\partial t^2}(\mathbf{c},t)+\frac{\mu}{r^3}\mathbf{r}(\mathbf{c},t)=\mathbf{0}$$

常数变易法

加速度可由速度方程对t求导得到

$$\ddot{\mathbf{r}}(\mathbf{c},t)=\frac{\partial^2\mathbf{r}(\mathbf{c},t)}{\partial t^2}+\frac{d\dot{\mathbf{r}}(\mathbf{c},t)}{d\mathbf{c}^T}\dot{\mathbf{c}}$$

$$\frac{d\dot{\mathbf{r}}(\mathbf{c},t)}{d\mathbf{c}^T}\dot{\mathbf{c}}=\frac{dR}{d\mathbf{r}}$$

带入含摄动项方程，得到

$$\frac{\partial^2\mathbf{r}(\mathbf{c},t)}{\partial t^2}+\frac{d\dot{\mathbf{r}}(\mathbf{c},t)}{d\mathbf{c}^T}\dot{\mathbf{c}}+\frac{\mu}{r^3}\mathbf{r}(\mathbf{c},t)=\frac{dR}{d\mathbf{r}}$$

$$\begin{bmatrix}\dfrac{d\mathbf{r}(\mathbf{c},t)}{d\mathbf{c}^T}\\ \dfrac{d\dot{\mathbf{r}}(\mathbf{c},t)}{d\mathbf{c}^T}\end{bmatrix}\dot{\mathbf{c}}=\begin{pmatrix}\mathbf{0}\\ \dfrac{dR}{d\mathbf{r}}\end{pmatrix}$$

常数变易法

$$\begin{bmatrix} -\dfrac{d\dot{\mathbf{r}}(\mathbf{c},t)}{d\mathbf{c}} & \dfrac{d\mathbf{r}(\mathbf{c},t)}{d\mathbf{c}} \end{bmatrix} \begin{bmatrix} \dfrac{d\mathbf{r}(\mathbf{c},t)}{d\mathbf{c}^T} \\ \dfrac{d\dot{\mathbf{r}}(\mathbf{c},t)}{d\mathbf{c}^T} \end{bmatrix} \dot{\mathbf{c}} = \begin{bmatrix} -\dfrac{d\dot{\mathbf{r}}(\mathbf{c},t)}{d\mathbf{c}} & \dfrac{d\mathbf{r}(\mathbf{c},t)}{d\mathbf{c}} \end{bmatrix} \begin{pmatrix} \mathbf{0} \\ \dfrac{dR}{d\mathbf{r}} \end{pmatrix}$$

$$\mathbf{L} = \left(\frac{\partial \mathbf{r}^T}{\partial c_i} \frac{\partial \dot{\mathbf{r}}}{\partial c_j} - \frac{\partial \mathbf{r}^T}{\partial c_j} \frac{\partial \dot{\mathbf{r}}}{\partial c_i} \right) = \left(\left[c_i, c_j \right] \right)$$

$$\mathbf{L}\dot{\mathbf{c}} = \frac{dR}{d\mathbf{c}}.$$

$$\left[c_i, c_j \right] = \frac{\partial \mathbf{r}^T}{\partial c_i} \frac{\partial \dot{\mathbf{r}}}{\partial c_j} - \frac{\partial \mathbf{r}^T}{\partial c_j} \frac{\partial \dot{\mathbf{r}}}{\partial c_i}$$

常数变易法

$$
\begin{pmatrix}
0 & 0 & -\frac{1}{2}na\sqrt{1-e^2} & -\frac{na}{2}\sqrt{1-e^2}\cos i & 0 & -\frac{an}{2} \\
0 & 0 & \frac{na^2e}{\sqrt{1-e^2}} & \frac{na^2e}{\sqrt{1-e^2}}\cos i & 0 & 0 \\
\frac{1}{2}na\sqrt{1-e^2} & -\frac{na^2e}{\sqrt{1-e^2}} & 0 & 0 & 0 & 0 \\
\frac{na}{2}\sqrt{1-e^2}\cos i & -\frac{na^2e}{\sqrt{1-e^2}}\cos i & 0 & 0 & -a^2n\sqrt{1-e^2}\sin i & 0 \\
0 & 0 & 0 & a^2n\sqrt{1-e^2}\sin i & 0 & 0 \\
\frac{an}{2} & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\frac{d}{dt}\begin{pmatrix} a \\ e \\ \omega \\ \Omega \\ i \\ M \end{pmatrix}
=
\begin{pmatrix} \frac{\partial}{\partial a} \\ \frac{\partial}{\partial e} \\ \frac{\partial}{\partial \omega} \\ \frac{\partial}{\partial \Omega} \\ \frac{\partial}{\partial i} \\ \frac{\partial}{\partial M} \end{pmatrix} R
$$

摄动方程

$$\frac{da}{dt}=\frac{2}{na}\frac{\partial R}{\partial M},$$

$$\frac{de}{dt}=\frac{1-e^2}{na^2e}\frac{\partial R}{\partial M}-\frac{\sqrt{1-e^2}}{na^2e}\frac{\partial R}{\partial \omega},$$

$$\frac{d\omega}{dt}=\frac{\sqrt{1-e^2}}{na^2e}\frac{\partial R}{\partial e}-\frac{\cot i}{na^2\sqrt{1-e^2}}\frac{\partial R}{\partial i},$$

$$\frac{d\Omega}{dt}=\frac{1}{na^2\sqrt{1-e^2}\sin i}\frac{\partial R}{\partial i},$$

$$\frac{di}{dt}=\frac{1}{na^2\sqrt{1-e^2}\sin i}\left(\cos i\frac{\partial R}{\partial \omega}-\frac{\partial R}{\partial \Omega}\right),$$

$$\frac{dM}{dt}=-\frac{1-e^2}{na^2e}\frac{\partial R}{\partial e}-\frac{2}{an}\frac{\partial R}{\partial a}.$$

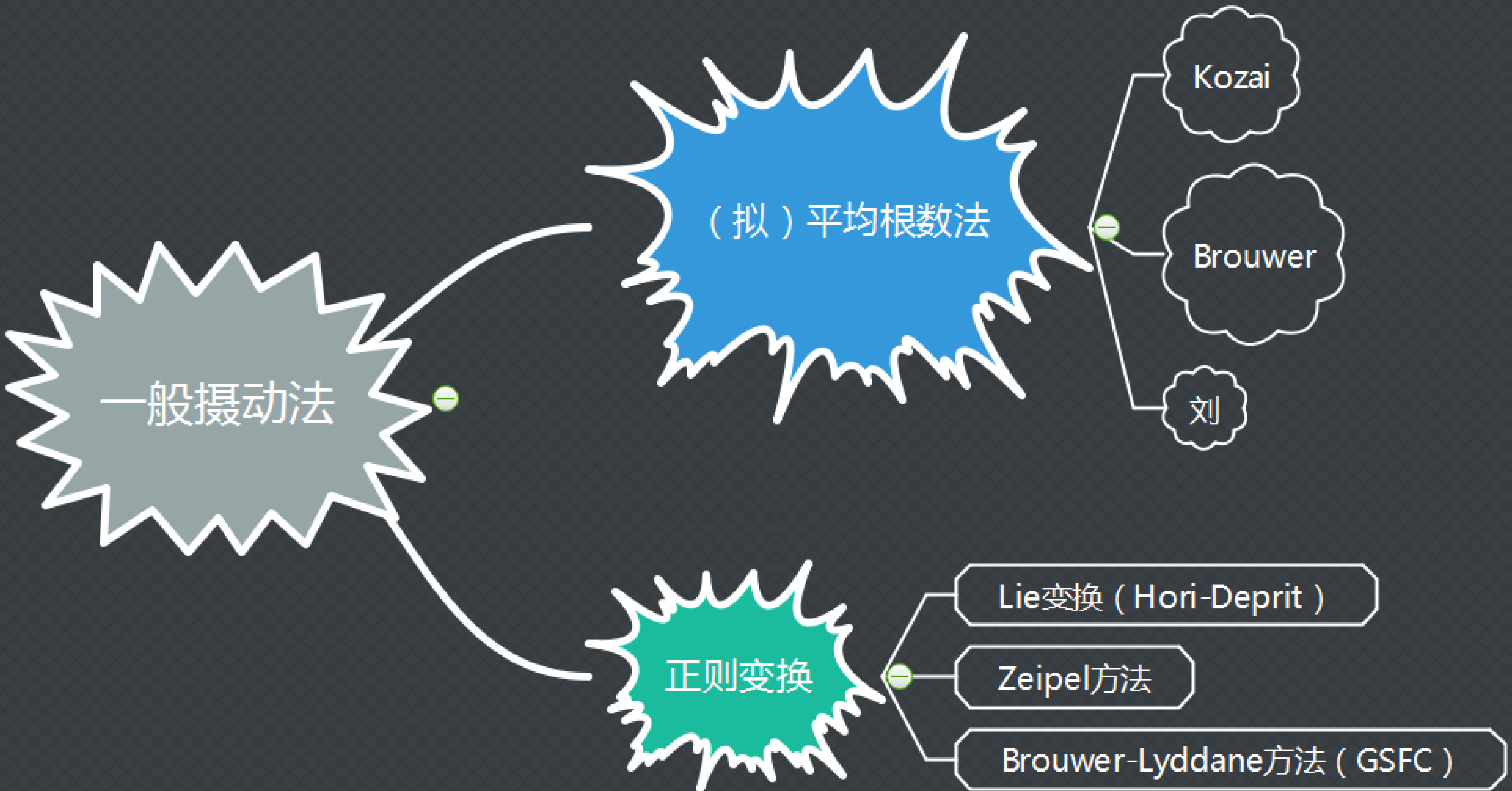
高斯型（S、T、W）

$$
\begin{cases}
\dfrac{da}{dt} = \dfrac{2}{n\sqrt{1-e^2}}\left[Se\sin f + T\left(1+e\cos f\right)\right] \\
\dfrac{de}{dt} = \dfrac{\sqrt{1-e^2}}{na}\left[S\sin f + T\left(\cos f + \cos E\right)\right] \\
\dfrac{di}{dt} = \dfrac{r\cos u}{na^2\sqrt{1-e^2}}W \\
\dfrac{d\Omega}{dt} = \dfrac{r\sin u}{na^2\sqrt{1-e^2}\sin i}W \\
\dfrac{d\omega}{dt} = \dfrac{\sqrt{1-e^2}}{nae}\left[-S\cos f + T\left(1+\dfrac{r}{p}\right)\sin f\right] - \cos i\dfrac{d\Omega}{dt} \\
\dfrac{dM}{dt} = n - \dfrac{1-e^2}{nae}\left[-S\left(\cos f - 2e\dfrac{r}{p}\right) + T\left(1+\dfrac{r}{p}\right)\sin f\right]
\end{cases}
$$

小偏心率小倾角高斯型根数摄动方程

$$
\begin{cases}
\dfrac{da}{dt}=\dfrac{2}{n\sqrt{1-e^2}}\left[S\left(\xi\sin u-\eta\cos u\right)+T\left(\dfrac{p}{r}\right)\right] \\
\dfrac{d\xi}{dt}=\dfrac{\sqrt{1-e^2}}{na}\left\{S\sin u+T\left[\left(\cos\tilde{u}+\cos u\right)-\dfrac{\eta}{\sqrt{1-e^2}\left(1+\sqrt{1-e^2}\right)}\left(\xi\sin\tilde{u}-\eta\cos\tilde{u}\right)\right]\right. \\
\qquad\qquad \left.+W\left(\dfrac{r}{p}\right)\left[-\dfrac{\eta}{\cos\left(i/2\right)}\left(h\sin u-k\cos u\right)\right]\right\} \\
\dfrac{d\eta}{dt}=\dfrac{\sqrt{1-e^2}}{na}\left\{-S\cos u+T\left[\left(\sin\tilde{u}+\sin u\right)+\dfrac{\xi}{\sqrt{1-e^2}\left(1+\sqrt{1-e^2}\right)}\left(\xi\sin\tilde{u}-\eta\cos\tilde{u}\right)\right]\right. \\
\qquad\qquad \left.+W\left(\dfrac{r}{p}\right)\left[\dfrac{\xi}{\cos\left(i/2\right)}\left(h\sin u-k\cos u\right)\right]\right\} \\
\dfrac{dh}{dt}=\dfrac{\sqrt{1-e^2}}{2na\cos\left(i/2\right)}W\left(\dfrac{r}{p}\right)\left[\cos u-h\left(h\cos u+k\sin u\right)\right] \\
\dfrac{dk}{dt}=\dfrac{\sqrt{1-e^2}}{2na\cos\left(i/2\right)}W\left(\dfrac{r}{p}\right)\left[\sin u-k\left(h\cos u+k\sin u\right)\right] \\
\dfrac{d\lambda}{dt}=n-\dfrac{\sqrt{1-e^2}}{na}\left\{2S\sqrt{1-e^2}\left(\dfrac{r}{p}\right)+\dfrac{1}{1+\sqrt{1-e^2}}\left[S\left(\xi\cos u+\eta\sin u\right)-T\left(1+\dfrac{r}{p}\right)\left(\xi\sin u-\eta\cos u\right)\right]\right. \\
\qquad\qquad \left.-W\left(\dfrac{r}{p}\right)\left(\dfrac{h\sin u-k\cos u}{\cos\left(i/2\right)}\right)\right\}
\end{cases}
$$

一般摄动法



一般摄动法
（拟）平均根数法
Kozai
Brouwer
刘
正则变换
Lie变换（Hori-Deprit）
Zeipel方法
Brouwer-Lyddane方法（GSFC）

摄动法构造（Liu）

$$\frac{d\sigma}{dt} = f_0(a) + f_1(\sigma, t, \varepsilon)$$

$$\begin{cases} f_0(a) = \delta n \\ \delta = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}^T \end{cases}$$

$$a^{(0)}(t) = a_0, \quad e^{(0)}(t) = e_0, \quad i^{(0)}(t) = i_0,$$
$$\Omega^{(0)}(t) = \Omega_0, \qquad \omega^{(0)}(t) = \omega_0$$
$$M^{(0)}(t) = M_0 + n_0(t - t_0)$$

$$\sigma(t) = \sigma^{(o)}(t) + \Delta\sigma^{(1)}(t, \varepsilon) + \Delta\sigma^{(2)}(t, \varepsilon^2) + \cdots + \Delta\sigma^{(l)}(t, \varepsilon^l) + \cdots$$

$$\Delta\sigma^{(l)}(t, \varepsilon^l) = \varepsilon^l \beta_l(t), \qquad l = 1, 2, \cdots$$

摄动法构造

$$\frac{d}{dt}\left[\sigma^{(o)}+\Delta\sigma^{(1)}+\Delta\sigma^{(2)}+\cdots+\Delta\sigma^{(l)}+\cdots\right]$$

$$= f_0(a)+\frac{\partial f_0}{\partial a}\left[\Delta a^{(1)}+\Delta a^{(2)}+\cdots\right]+\frac{1}{2}\frac{\partial^2 f_0}{\partial a^2}\left[\Delta a^{(1)}+\cdots\right]^2+\cdots$$

$$+f_1(\sigma,t,\varepsilon)+\sum_{j=1}^{6}\frac{\partial f_1}{\partial\sigma_j}\left[\Delta\sigma_j^{(1)}+\Delta\sigma_j^{(2)}+\cdots\right]$$

$$+\frac{1}{2}\sum_{j=1}^{6}\sum_{k=1}^{6}\frac{\partial^2 f_1}{\partial\delta_j\partial\delta_k}\left[\Delta\sigma_j^{(1)}+\cdots\right]\left[\Delta\sigma_k^{(1)}+\cdots\right]+\cdots$$

$$\begin{cases}\sigma^{(o)}(t)=\sigma_0+\delta n_0(t-t_0)\\ \Delta\sigma^{(1)}(t)=\int_{t_0}^{t}\left[\delta\frac{\partial n}{\partial a}\Delta a^{(1)}+f_1(\sigma,t,\varepsilon)_1\right]_{\sigma^{(o)}}dt\\ \Delta\sigma^{(2)}(t)=\int_{t_0}^{t}\left[\delta\left(\frac{\partial n}{\partial a}\Delta a^{(2)}+\frac{1}{2}\frac{\partial^2 n}{\partial a^2}\left(\Delta a^{(1)}\right)^2\right)+\sum_j\frac{\partial f_1}{\partial\sigma_j}\Delta\sigma_j^{(1)}\right]_{\sigma^{(o)}}dt\\ \cdots\end{cases}$$

摄动法范例（Liu Lin）

$$\ddot{x} + \omega^2 x = -\varepsilon x^3, \quad \varepsilon \ll 1.$$

无摄运动方程

$$\ddot{x} + \omega^2 x = 0$$

$$\begin{cases} x = a\cos(\omega t + M_0), \\ \dot{x} = -\omega a \sin(\omega t + M_0). \end{cases}$$

常数变易法建立摄动方程

$$\begin{cases} \dfrac{\partial x}{\partial a}\dot{a} + \dfrac{\partial x}{\partial M_0}\dot{M}_0 = 0, \\ \dfrac{\partial \dot{x}}{\partial a}\dot{a} + \dfrac{\partial \dot{x}}{\partial M_0}\dot{M}_0 = -\varepsilon x^3. \end{cases}$$

$$\begin{cases} \dot{a} = \dfrac{\varepsilon}{\omega}a^3\left(\dfrac{1}{4}\sin 2M + \dfrac{1}{8}\sin 4M\right) = (f_1)_a, \\ \dot{M} = \omega + \dot{M}_0 = \omega + \dfrac{\varepsilon}{\omega}a^2\left(\dfrac{3}{8} + \dfrac{1}{2}\cos 2M + \dfrac{1}{8}\cos 4M\right) = \omega + (f_1)_M. \end{cases}$$

摄动法范例

其中积分常数 M_0 用 $M=M_0+\omega t$ 代替，方程　　　的小参数幂级数解即

$$\sigma(t) = \sigma^{(0)}(t) + \Delta\sigma^{(1)}(t) + \cdots .$$

其中

$$\sigma^{(0)}(t) = \begin{pmatrix} a_0 \\ M_0 + \omega t \end{pmatrix} .$$

因 $\omega=\text{const}$，于是有

$$\Delta\sigma^{(1)}(t) = \int_0^t [f_1(\sigma,t,\varepsilon)]_{\sigma^{(0)}} \mathrm{d}t .$$

摄动法范例

$$\Delta a^{(1)}(t) = \frac{\varepsilon}{\omega^2} a^3 \left(-\frac{1}{8}\cos 2M - \frac{1}{32}\cos 4M\right)\Bigg|_0^t,$$

$$\Delta M^{(1)}(t) = \frac{\varepsilon}{\omega^2} a^2 \left(\frac{3}{8}\omega t + \frac{1}{4}\sin 2M + \frac{1}{32}\sin 4M\right)\Bigg|_0^t.$$

二阶摄动项的计算公式为

$$\begin{cases} \Delta a^{(2)}(t) = \int_0^t \left[\frac{\partial (f_1)_a}{\partial a}\Delta a^{(1)} + \frac{\partial (f_1)_a}{\partial M}\Delta M^{(1)}\right]_{\sigma^{(0)}} \mathrm{d}t, \\ \Delta M^{(2)}(t) = \int_0^t \left[\frac{\partial (f_1)_M}{\partial a}\Delta a^{(1)} + \frac{\partial (f_1)_M}{\partial M}\Delta M^{(1)}\right]_{\sigma^{(0)}} \mathrm{d}t. \end{cases}$$

将 $\Delta\sigma^{(1)}$ 代入后积分即得二阶摄动项 $\Delta a^{(2)}$ 和 $\Delta M^{(2)}$. 不难看出，由于 $\Delta M^{(1)}$ 中含有 ωt 这种项，那么求 $\Delta\sigma^{(2)}(t)$ 时，将会出现下列形式的积分：

$$\int_0^t \begin{pmatrix} \sin kM \\ \cos kM \end{pmatrix} \omega t\,\mathrm{d}t, \quad k = 0, 1, \cdots.$$

此即混合项，亦称泊松(Poisson)项，正是摄动法在动力天文中用来求解摄动运动方程时应重视的问题.

级数方法摄动法步骤（林家翘）

假定因变量可以展为小参数的幂级数

- 第一步：把幂级数带入微分方程
- 第二步：把所有量展开，使得每一项都可以写为一个幂级数。
- 第三步：把方程中所有同幂次项收集起来，并让级数中各幂次的系数等于零。
- 第四步：把幂级数代入原先的边界或初始条件，展开系数等于零。这样就得到一组隶属于第三步所得到的一系列微分方程的初值或边界条件。
- 第五步：相继求解第一步至第四步的一系列微分方程和边界条件。

常微分方程主要摄动法（林家翘）

- 级数方法
- 参数微商法
- 逐次逼近法
- ……

当函数满足
◆代数方程
◆差分方程
◆偏微分方程
◆积分方程
等情况时，摄动法思想与手段是类似的。

拟平均根数法（Liu）

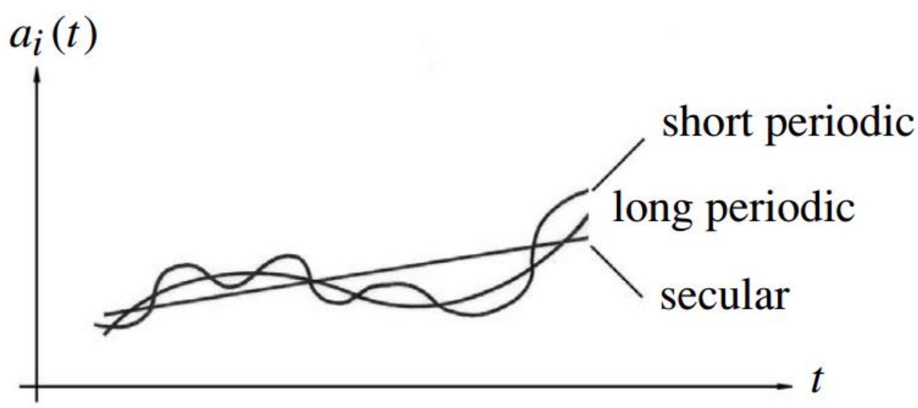
拟平均根数定义

$$\bar{\sigma} = \bar{\sigma}_0 + (\sigma_1 + \sigma_2)(t - t_0) + \left[\sigma_l^{(1)}(t) - \sigma_l^{(1)}(t_0)\right]$$

$$\bar{\sigma}_0 = \sigma_0 - \left[\sigma_s^{(1)}(t_0) + \sigma_s^{(2)}(t_0)\right]$$

选择无奇点根数

$$\sigma = (a, i, \Omega, \xi = e\cos\omega, \eta = -e\sin\omega, \lambda = M + \omega)$$



计算步骤

1） 由$\bar{\sigma}_0$计算$\sigma_1(t-t_0)$，并由下式定义的平根数给出计算中用到的$\bar{\sigma}$：

$$\left.\begin{aligned}
&\bar{a} = \bar{a}_0 \qquad \bar{i} = \bar{i}_0 \\
&\bar{\Omega} = \bar{\Omega}_0 + \Omega_1(t - t_0) \\
&\bar{\xi} = \bar{\xi}_0 \cos\omega_1(t - t_0) + \bar{\eta}_0 \sin\omega_1(t - t_0) \\
&\bar{\eta} = \bar{\eta}_0 \cos\omega_1(t - t_0) - \bar{\xi}_0 \sin\omega_1(t - t_0) \\
&\bar{\lambda} = \bar{\lambda}_0 + (\bar{n}_0 + \lambda_1)(t - t_0)
\end{aligned}\right\}$$

2） 由$\bar{\sigma}_0$和$\bar{\sigma}$计算$\sigma_2(t-t_0)$和$\left[\sigma_l^{(1)}(t) - \sigma_l^{(1)}(t_0)\right]$；

3） 由$\bar{\sigma}$、$\sigma_2(t-t_0)$和$\left[\sigma_l^{(1)}(t) - \sigma_l^{(1)}(t_0)\right]$计算出真正的平根数$\bar{\sigma}$，即：

$$\bar{\sigma} = \bar{\sigma}_0 + (\sigma_1 + \sigma_2)(t - t_0) + \left[\sigma_l^{(1)}(t) - \sigma_l^{(1)}(t_0)\right]$$

4） 由$\bar{\sigma}$计算$\sigma_s^{(1)}(t), \sigma_s^{(2)}(t)$，最后给出瞬根数$\sigma = \bar{\sigma} + \sigma_s^{(1)}(t) + \sigma_s^{(2)}(t)$。

一阶长期项

$$\left.\begin{aligned}
&a_1 = e_1 = i_1 = 0 \\
&\Omega_1 = -\frac{3}{2}\frac{J_2}{P^2}n\cos i \\
&\omega_1 = \frac{3}{4}\frac{J_2}{P^2}n\left(4 - 5\sin^2 i\right) \\
&\lambda_1 = \frac{3}{4}\frac{J_2}{P^2}n\left[\left(2 - 3\sin^2 i\right)\sqrt{1 - e^2} + \left(4 - 5\sin^2 i\right)\right]
\end{aligned}\right\}$$

一阶短周期项

$$a_s^{(1)} = \frac{J_2}{a}\left\{ \left(1-\frac{3}{2}sin^2 i\right)\left[\left(\frac{a}{r}\right)^3 - \left(1-e^2\right)^{-\frac{3}{2}}\right] + \frac{3}{2}\left(\frac{a}{r}\right)^3 sin^2 i\, cos\, 2u \right\}$$

$$i_s^{(1)} = \frac{J_2}{8P^2} sin\, 2i\{3\, cos\, 2u + 3(\xi\, cos\, u + \eta\, sin\, u)$$

$$+ (\xi\, cos\, 3u - \eta\, sin\, 3u) + \left(\xi^2 - \eta^2\right)F_2(e)\}$$

$$\Omega_s^{(1)} = \frac{3J_2}{2P^2} \cos i\left\{-(u-\lambda) + \frac{1}{2}\sin 2u - \frac{1}{2}(\xi \sin u + 3\eta \cos u)\right.$$

$$\left. + \frac{1}{6}(\xi \sin 3u + \eta \cos 3u) - \frac{1}{3}\xi\eta F_2(e)\right\}$$

一阶短周期项

$$\xi_s^{(1)} = \frac{3}{2}\frac{J_2}{P^2}\left\{\left(2-\frac{5}{2}\sin^2 i\right)(u-\lambda)\eta+\left(1+\frac{1}{4}\xi^2+\frac{9}{4}\eta^2\right)\cos u\right.$$

$$+\xi\eta\sin u+\frac{1}{2}\xi\cos 2u-\eta\sin 2u+\frac{1}{12}\left(\xi^2-3\eta^2\right)\cos 3u$$

$$-\frac{1}{3}\xi\eta\sin 3u+\left(1-\frac{e^2}{12}F_2(e)\right)\xi-\frac{1}{12}F_2(e)\left(\xi^2-3\eta^2\right)\xi$$

$$+\sin^2 i\left[\left(-\frac{5}{4}+\frac{3}{8}\xi^2-\frac{25}{8}\eta^2\right)\cos u-\frac{1}{4}\xi\eta\sin u\right.$$

$$+\frac{1}{2}\xi\cos 2u+2\eta\sin 2u+\left(\frac{7}{12}+\frac{11}{48}\xi^2+\frac{25}{48}\eta^2\right)\cos 3u$$

$$+\frac{7}{24}\xi\eta\sin 3u+\frac{3}{8}\xi\cos 4u-\frac{3}{8}\eta\sin 4u+\frac{1}{16}\left(\xi^2-\eta^2\right)\cos 5u$$

$$\left.\left.-\frac{1}{8}\xi\eta\sin 5u-\left(\frac{5}{4}-\frac{e^2}{6}F_2(e)\right)\xi+\left(\frac{F_2(e)}{4}-\frac{F_3(e)}{6}\right)\left(\xi^2-3\eta^2\right)\xi\right]\right\}$$

一阶短周期项

$$\eta_s^{(1)} = \frac{3}{2}\frac{J_2}{P^2}\Bigg\{-\left(2-\frac{5}{2}\sin^2 i\right)(u-\lambda)\xi-\left(1+\frac{5}{4}\xi^2+\frac{1}{4}\eta^2\right)\sin u$$

$$-2\xi\eta\cos u-\frac{1}{2}\eta\cos 2u+\frac{1}{12}e^2\sin 3u+$$

$$\left(1-\frac{e^2}{12}F_2(e)\right)\eta+\frac{1}{12}F_2(e)\left(\eta^2-3\xi^2\right)\eta$$

$$+\sin^2 i\Bigg[\left(\frac{7}{4}+\frac{9}{8}\xi^2+\frac{9}{8}\eta^2\right)\sin u+\frac{13}{4}\xi\eta\cos u-\frac{1}{2}\xi\sin 2u$$

$$+2\eta\cos 2u-\left(\frac{7}{12}+\frac{13}{48}\xi^2+\frac{23}{48}\eta^2\right)\sin 3u$$

$$+\frac{5}{24}\xi\eta\cos 3u-\frac{3}{8}\eta\cos 4u-\frac{3}{8}\xi\sin 4u-\frac{1}{16}\left(\xi^2-\eta^2\right)\sin 5u$$

$$-\frac{1}{8}\xi\eta\cos 5u-\left(\frac{5}{4}-\frac{e^2}{6}F_2(e)\right)\eta+\left(\frac{F_2(e)}{4}-\frac{F_3(e)}{6}\right)\left(3\xi^2-\eta^2\right)\eta\Bigg]\Bigg\}$$

一阶短周期项

$$\lambda_s^{(1)} = \frac{3}{2}\frac{J_2}{P^2}\left\{\left(2-\frac{5}{2}\sin^2 i\right)(u-\lambda)-\frac{1}{2}\sin 2u+\frac{3}{2}\xi\sin u+\frac{5}{2}\eta\cos u\right.$$

$$-\frac{1}{6}(\xi\sin 3u+\eta\cos 3u)+\frac{1}{1+\sqrt{1-e^2}}\left[\left(1-\frac{1}{4}e^2\right)(\xi\sin u+\eta\cos u)\right.$$

$$+\frac{1}{2}\left(\xi^2-\eta^2\right)\sin 2u+\xi\eta\cos 2u+\frac{1}{12}\left(\xi^2-3\eta^2\right)\xi\sin 3u$$

$$\left.\left.+\frac{1}{12}\left(3\xi^2-\eta^2\right)\eta\cos 3u\right]+\frac{1}{3}\xi\eta F_2(e)\right\}$$

$$+\frac{3}{2}\frac{J_2}{P^2}\sin^2 i\left\{\frac{5}{4}\sin 2u-\frac{5}{2}(\xi\sin u+\eta\cos u)-\frac{5}{6}\xi\eta F_2(e)\right.$$

$$+\frac{1}{1+\sqrt{1-e^2}}\left[\left(-\frac{1}{2}+\frac{5}{4}\sqrt{1-e^2}+\frac{1}{8}\left(\xi^2-\eta^2\right)\right)\xi\sin u\right.$$

$$+\left(-\frac{5}{2}-\frac{5}{4}\sqrt{1-e^2}+\frac{1}{8}\left(7\xi^2+5\eta^2\right)\right)\eta\cos u-\frac{3}{2}\xi\eta\cos 2u$$

$$-\frac{3}{4}\left(\xi^2-\eta^2\right)\sin 2u+\left(1+\frac{5}{12}\sqrt{1-e^2}-\frac{7}{48}\xi^2+\frac{17}{48}\eta^2\right)\xi\sin 3u$$

$$\left(1+\frac{5}{12}\sqrt{1-e^2}-\frac{19}{48}\xi^2+\frac{5}{48}\eta^2\right)\eta\cos 3u+\frac{3}{8}\left(\xi^2-\eta^2\right)\sin 4u$$

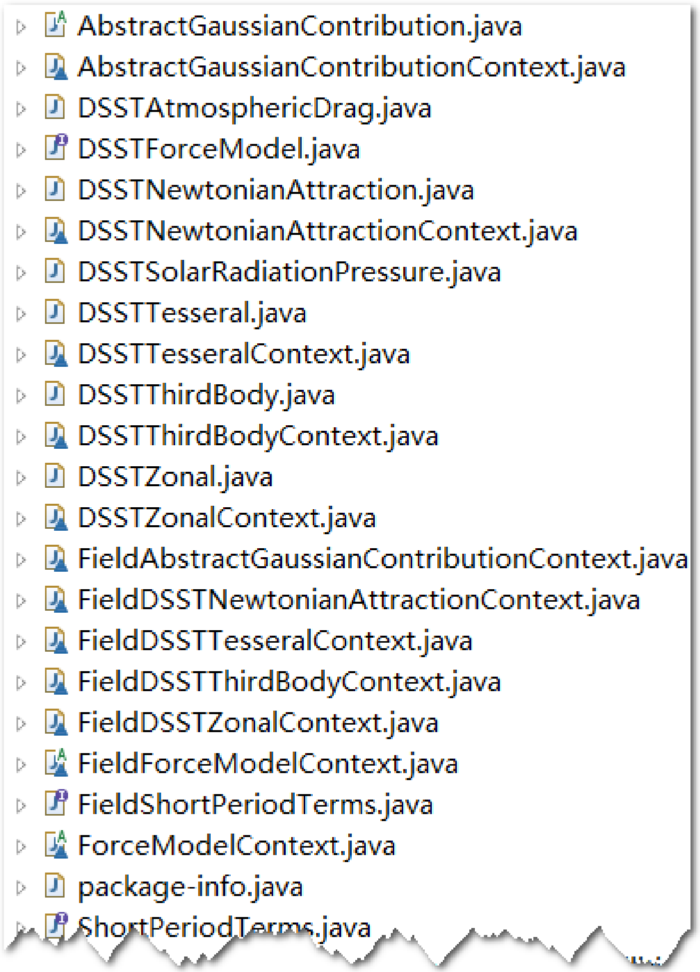
$$+\frac{3}{4}\xi\eta\cos 4u+\frac{1}{16}\left(\xi^2-3\eta^2\right)\xi\sin 5u+\frac{1}{16}\left(3\xi^2-\eta^2\right)\eta\cos 5u$$

$$\left.\left.+\left(\frac{1}{4}+\left(\frac{1}{3}+\frac{e^2}{6}\right)F_2(e)\right)\xi\eta\right]\right\}$$

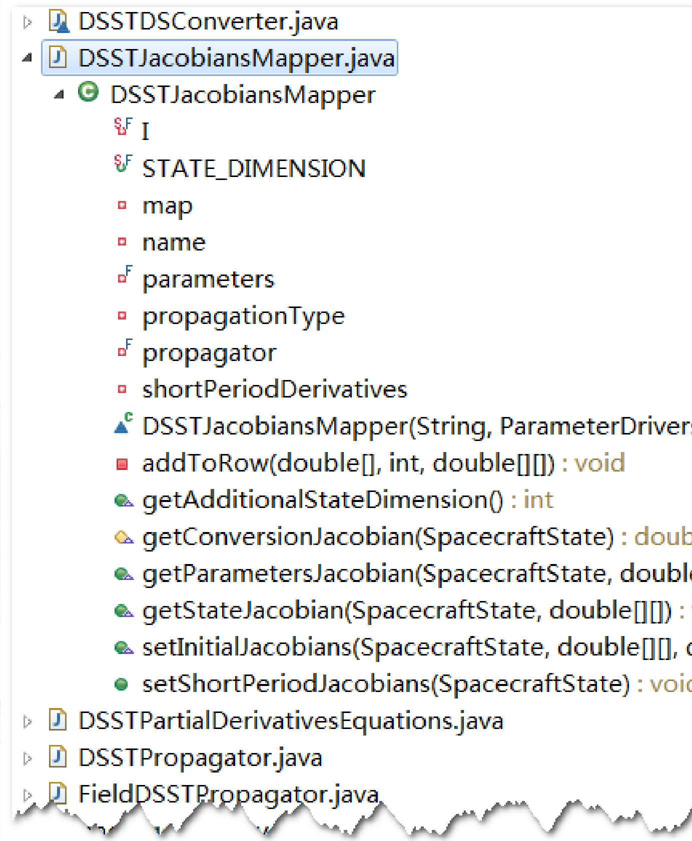
$$F_2(e)=\frac{\overline{\cos 2f}}{e^2},\quad F_3(e)=\frac{\overline{\cos 2f}-3e^2/4}{e^4}$$

一阶长周期项及二阶项略，参考文献：
《航天器轨道理论》，刘林，
国防工业出版社，2000

Draper Semianalytical Satellite Theory (DSST) 半解析算法



动力学模型



偏导数部分

半分析带谐项解

$$K_{n1}(e) = \sum_{\substack{l=0 \\ \text{step } 2}}^{n-2} \binom{n-1}{l} \binom{l}{l/2} \left(\frac{e}{2}\right)^l$$

$$\frac{l}{e^2} K_{n1}(e) = \sum_{\substack{l=0 \\ \text{step } 2}}^{n-2} \frac{l}{4} \binom{n-1}{l} \binom{l}{l/2} \left(\frac{e}{2}\right)^{l-2}$$

$$\dot{a}_{sec} = \dot{e}_{sec} = \mathrm{d}i/\mathrm{d}t_{\,sec} = 0$$

$$\dot{\Omega}_{sec} = n\cos i \sum_{\substack{n\geq 2 \\ \text{step } 2}} -J_n \left(\frac{r_e}{p}\right)^n K_{n1}(e) \sum_{q=1}^{n/2} (-1)^{\frac{1}{2}(n+2q)} \frac{2q}{2^{n+2q}} \binom{n}{\frac{n}{2}-q} \binom{n+2q}{n} \binom{2q}{q} (\sin i)^{2q-2}$$

$$\dot{\omega}_{sec} = [\dot{\omega}]_c - \cos i \cdot \dot{\Omega}_{sec}$$

$$[\dot{\omega}]_c = n \sum_{\substack{n\geq 2 \\ \text{step } 2}} -J_n \left(\frac{r_e}{p}\right)^n \left[\sum_{q=0}^{n/2} (-1)^{\frac{1}{2}(n+2q)} 2^{-(n+2q)} \binom{n}{\frac{n}{2}-q} \binom{n+2q}{n} \binom{2q}{q} (\sin i)^{2q}\right]$$

$$\cdot \left[(1-e^2)\left(\frac{l}{e^2} K_{n1}(e)\right) + (2n-1) K_{n1}(e)\right]$$

$$\dot{M}_{sec} = n\sqrt{1-e^2} \sum_{\substack{n\geq 2 \\ \text{step } 2}} -J_n \left(\frac{r_e}{p}\right)^n K_{n1}(e)$$

$$\cdot \sum_{q=0}^{n/2} (-1)^{\frac{1}{2}(n+2q)} \frac{2(n+1)}{2^{n+2q}} \binom{n}{\frac{n}{2}-q} \binom{n+2q}{n} \binom{2q}{q} (\sin i)^{2q} - \sqrt{1-e^2}[\dot{\omega}]_c$$

半分析长期项

$$\dot{a}_{long} = 0$$

$$\dot{e}_{long} = -\left(\frac{1-e^2}{e}\tan i\right) \mathrm{d}i/\mathrm{d}t_{\,long}$$

$$\mathrm{d}i/\mathrm{d}t_{\,long} = n\cos i \sum_{n\geq 3} -J_n \left(\frac{r_e}{p}\right)^n \sum_{p=0}^{(n-2+\delta_1)/2} K_{n1}^p(e)\,(n-2p)$$

$$\times \left[-(1-\delta_1)\sin(n-2p)\,\omega + \delta_1 \cos(n-2p)\,\omega\right]$$

$$\times \left[\sum_{q=0}^{p} (-1)^{\frac{1}{2}(n+2q-\delta_1)} 2^{-(2n-2p+2q-1)} \binom{n}{p-q} \binom{2n-2p+2q}{n} \binom{n-2p+2q}{q} (\sin i)^{n-2p+2q-1}\right]$$

$$\dot{\Omega}_{long} = n\cos i \sum_{n\geq 3} -J_n \left(\frac{r_e}{p}\right)^n \sum_{p=0}^{(n-2+\delta_1)/2} K_{n1}^p(e)\left[(1-\delta_1)\cos(n-2p)\,\omega + \delta_1 \sin(n-2p)\,\omega\right]$$

$$\times \left[\sum_{q=0}^{p} (-1)^{\frac{1}{2}(n+2q-\delta_1)} \frac{(n-2p+2q)}{2^{(2n-2p+2q-1)}} \binom{n}{p-q} \binom{2n-2p+2q}{n} \binom{n-2p+2q}{q} (\sin i)^{n-2p+2q-2}\right]$$

$$\dot{\omega}_{long} = [\dot{\omega}]_l - \cos i \times \dot{\Omega}_{long}$$

半分析长期项

$$\delta_1 = \frac{1}{2}\left[1-(-1)^n\right],\ \delta_2 = \begin{cases} 0 & p=0 \\ 1 & p\neq 0 \end{cases}$$

$$[\dot{\omega}]_l = n\sum_{n\geq 3} -J_n\left(\frac{r_e}{p}\right)^n$$

$$\times \sum_{p=0}^{(n-2+\delta_1)/2}\left[\left(1-e^2\right)\left(\frac{l}{e^2}K_{n1}^p(e)\right)+(2n-1)K_{n1}^p(e)\right]$$

$$\times\left[(1-\delta_1)\cos(n-2p)\omega+\delta_1\sin(n-2p)\omega\right]$$

$$\times\left[\sum_{q=0}^{p}(-1)^{\frac{1}{2}(n+2q-\delta_1)}2^{-(2n-2p+2q-1)}\binom{n}{p-q}\binom{2n-2p+2q}{n}\binom{n-2p+2q}{q}(\sin i)^{n-2p+2q}\right]$$

$$K_{n1}^p(e) = \delta_2\sum_{\substack{l=n-2p\\ \text{step } 2}}^{n-2}\binom{n-1}{l}\binom{l}{\frac{1}{2}(l-n+2p)}\left(\frac{e}{2}\right)^l$$

$$\frac{l}{e^2}K_{n1}^p(e) = \delta_2\sum_{\substack{l=n-2p\\ \text{step } 2}}^{n-2}\frac{l}{4}\binom{n-1}{l}\binom{l}{\frac{1}{2}(l-n+2p)}\left(\frac{e}{2}\right)^{l-2}$$

$$\dot{M}_{long} = -\sqrt{1-e^2}[\dot{\omega}]_l+n\sqrt{1-e^2}\sum_{n\geq 3}-J_n\left(\frac{r_e}{p}\right)^n$$

$$\times\sum_{p=0}^{(n-2+\delta_1)/2}K_{n1}^p(e)\left[(1-\delta_1)\cos(n-2p)\omega+\delta_1\sin(n-2p)\omega\right]$$

$$\times\left[\sum_{q=0}^{p}(-1)^{\frac{1}{2}(n+2q-\delta_1)}\frac{(n+1)}{2^{(2n-2p+2q-2)}}\binom{n}{p-q}\binom{2n-2p+2q}{n}\binom{n-2p+2q}{q}(\sin i)^{n-2p+2q}\right]$$

二阶影响

$$\dot{a}_{long,22} = \dot{e}_{long,22} = \dot{a}_{long,31} = \dot{a}_{long,32} = \dot{a}_{long,33} = 0$$

$$di/dt_{\ long,22} = 3\left(\frac{r_e}{p}\right)^2 n \sin i \left(C_{22} \sin 2\Omega_s - S_{22} \cos 2\Omega_s\right)$$

$$\dot{\Omega}_{long,22} = 3\left(\frac{r_e}{p}\right)^2 n \cos i \left(C_{22} \cos 2\Omega_s + S_{22} \sin 2\Omega_s\right)$$

$$\dot{\omega}_{long,22} = \frac{3}{2}\left(\frac{r_e}{p}\right)^2 n \left(5\sin^2 i - 2\right)\left(C_{22} \cos 2\Omega_s + S_{22} \sin 2\Omega_s\right)$$

$$\dot{M}_{long,22} = \frac{9}{2}\left(\frac{r_e}{p}\right)^2 n\sin^2 i(1-e^2)^{\frac{1}{2}}\left(C_{22} \cos 2\Omega_s + S_{22} \sin 2\Omega_s\right)$$

$$\begin{aligned}\dot{e}_{long,31} = & \frac{3}{8}\left(\frac{r_e}{p}\right)^3 n\left(1-e^2\right) \\ & \times \{C_{31}\left[\sin\omega \cos\Omega_s \left(5\sin^2 i - 4\right) + \cos i \cos\omega \sin\Omega_s \left(15\sin^2 i - 4\right)\right] \\ & +S_{31}\left[\sin\omega \sin\Omega_s \left(5\sin^2 i - 4\right) + \cos i \cos\omega \cos\Omega_s \left(4 - 15\sin^2 i\right)\right]\}\end{aligned}$$

$$\begin{aligned}di/dt_{\ long,31} = & \frac{3}{8}\left(\frac{r_e}{p}\right)^3 ne \sin i \{C_{31}\left[10\cos i \sin\omega \cos\Omega_s + \cos\omega \sin\Omega_s \left(1 - 15\cos^2 i\right)\right] \\ & +S_{31}\left[10\cos i \sin\omega \sin\Omega_s + \cos\omega \cos\Omega_s \left(15\cos^2 i - 1\right)\right]\}\end{aligned}$$

二阶影响

$$\dot{\Omega}_{long,31} = \frac{3}{8}\left(\frac{r_e}{p}\right)^3 ne\left\{C_{31}\left[10\cos i\cos\omega\cos\Omega_s + \sin\omega\sin\Omega_s\left(11 - 45\cos^2 i\right)\right]\right.$$
$$\left.+S_{31}\left[10\cos i\cos\omega\sin\Omega_s + \sin\omega\cos\Omega_s\left(45\cos^2 i - 11\right)\right]\right\}$$

$$\dot{\omega}_{long,31} = -\frac{3}{8}\left(\frac{r_e}{p}\right)^3 n\left(4e + \frac{1}{e}\right)$$
$$\times\left\{C_{31}\left[\cos\omega\cos\Omega_s\left(4 - 5\sin^2 i\right) + \cos i\sin\omega\sin\Omega_s\left(15\sin^2 i - 4\right)\right]\right.$$
$$\left.+S_{31}\left[\cos\omega\sin\Omega_s\left(4 - 5\sin^2 i\right) + \cos i\sin\omega\cos\Omega_s\left(4 - 15\sin^2 i\right)\right]\right\}$$
$$-\dot{\Omega}_{31}\cos i$$

$$\dot{M}_{long,31} = \frac{3}{8}\left(\frac{r_e}{p}\right)^3 n(1 - e^2)^{\frac{1}{2}}$$
$$\times\left\{C_{31}\left[\cos\Omega_s\cos\omega\left(5\sin^2 i - 4\right) + \sin\Omega_s\sin\omega\cos i\left(4 - 15\sin^2 i\right)\right]\right.$$
$$\left.+S_{31}\left[\sin\Omega_s\cos\omega\left(5\sin^2 i - 1\right) + \cos\Omega_s\sin\omega\cos i\left(15\sin^2 i - 4\right)\right]\right\}$$
$$\times\left(4e - \frac{1}{e}\right)$$

二阶影响

$$\dot{e}_{long,32} = -\frac{15}{4}\left(\frac{r_e}{p}\right)^3 n\left(1-e^2\right)\sin^2 i$$

$$\times\left\{C_{32}\left[\cos\omega\cos 2\Omega_s\left(3\sin^2 i-2\right)+2\cos i\sin\omega\sin 2\Omega_s\right]\right.$$

$$\left.+S_{32}\left[\cos\omega\sin 2\Omega_s\left(3\sin^2 i-2\right)-2\cos i\sin\omega\cos\Omega_s\right]\right\}$$

$$di/dt_{\ long,32} = -\frac{15}{4}\left(\frac{r_e}{p}\right)^3 ne$$

$$\times\left\{C_{32}\left[\sin\omega\sin 2\Omega_s\left(2-4\sin^2 i\right)-\cos i\cos\omega\cos 2\Omega_s\left(2+3\sin^2 i\right)\right]\right.$$

$$\left.+S_{32}\left[\sin\omega\cos 2\Omega_s\left(4\sin^2 i-2\right)-\cos i\cos\omega\sin 2\Omega_s\left(2+3\sin^2 i\right)\right]\right\}$$

$$\dot{e}_{long,33} = \frac{45}{4}\left(\frac{r_e}{p}\right)^3 n\left(1-e^2\right)\left\{C_{33}\left[2\cos 3\Omega_s\sin\omega\cos i-\sin 3\Omega_s\cos\omega\left(3\sin^2 i-2\right)\right]\right.$$

$$\left.+S_{33}\left[2\sin 3\Omega_s\sin\omega\cos i-\cos 3\Omega_s\cos\omega\left(2-3\sin^2 i\right)\right]\right\}$$

$$di/dt_{\ long,33} = \frac{45}{4}\left(\frac{r_e}{p}\right)^3 n\frac{e}{\sin i}$$

$$\times\left\{C_{33}\left[\cos 3\Omega_s\sin\omega\left(4-7\sin^2 i\right)+\sin 3\Omega_s\cos\omega\cos i\left(4+3\sin^2 i\right)\right]\right.$$

$$\left.+S_{33}\left[\sin 3\Omega_s\sin\omega\left(4-7\sin^2 i\right)-\cos 3\Omega_s\cos\omega\cos i\left(4+3\sin^2 i\right)\right]\right\}$$

$$\dot{\Omega}_{long,33} = \frac{45}{2}\left(\frac{r_e}{p}\right)^3 ne\left[C_{33}\left(3\sin 3\Omega_s\sin\omega\cos i-\cos 3\Omega_s\cos\omega\right)\right.$$

$$\left.-S_{33}\left(3\cos 3\Omega_s\sin\omega\cos i-\sin 3\Omega_s\cos\omega\right)\right]$$

二阶影响

$$\dot{\omega}_{long,33}=\frac{45}{4}\left(\frac{r_e}{p}\right)^3 n\left(4e+\frac{1}{e}\right)\{C_{33}\left[2\cos 3\Omega_s\cos\omega\cos i+\sin 3\Omega_s\sin\omega\left(3\sin^2 i-2\right)\right]$$

$$+S_{33}\left[2\sin 3\Omega_s\cos\omega\cos i+\cos 3\Omega_s\sin\omega\left(2-3\sin^2 i\right)\right]\}-\dot{\Omega}_{33}\cos i$$

$$\dot{M}_{long,33}=\frac{45}{4}\left(\frac{r_e}{p}\right)^3 n(1-e^2)^{\frac{1}{2}}\left(4e-\frac{1}{e}\right)$$

$$\times\{C_{33}\left[2\cos 3\Omega_s\cos\omega\cos i+\sin 3\Omega_s\sin\omega\left(3\sin^2 i-2\right)\right]$$

$$+S_{33}\left[2\sin 3\Omega_s\cos\omega\cos i+\cos 3\Omega_s\sin\omega\left(2-3\sin^2 i\right)\right]\}$$

$$\tan m\lambda_{nm}=\frac{S_{nm}}{C_{nm}}$$

$$J_{nm}=-\left(C_{nm}^2+S_{nm}^2\right)^{\frac{1}{2}}$$

扇谐与田谐

$$a_{short,22} = -\frac{3}{2}\frac{J_{22}}{a}r_e^2$$

$$\times\left\{(1+\cos i)^2\left[\frac{1}{1-\tau}\cos(2M+2\omega+2\Omega_e)-\frac{1}{2}e\left(\frac{1}{1-2\tau}\cos(M+2\omega+2\Omega_e)\right.\right.\right.$$

$$\left.\left.-\frac{21}{3-2\tau}\cos(3M+2\omega+2\Omega_e)\right)\right]+(1-\cos i)^2\left[\frac{1}{1+\tau}\cos(2m+2\omega-2\Omega_e)\right.$$

$$\left.-\frac{e}{2}\left(\frac{1}{1+2\tau}\cos(M+2\omega-2\Omega_e)-\frac{21}{3-2\tau}\cos(3M+2\omega-2\Omega_e)\right)\right]$$

$$\left.+3e\sin^2 i\left[\frac{1}{1-2\tau}\cos(M+2\Omega_e)+\frac{1}{1+2\tau}\cos(M-2\Omega_e)\right]\right\}$$

扇谐与田谐

$$
\begin{aligned}
e_{short,22} = &-\frac{3}{4}J_{22}\left(\frac{r_e}{a}\right)^2 \\
&\times\left\{(1+\cos i)^2\left[\frac{1}{2-4\tau}\cos\left(M+2\omega+2\Omega_e\right)+\frac{7}{6-4\tau}\cos\left(3M+2\omega+2\Omega_e\right)\right.\right. \\
&\left.+\frac{e}{2}\left(\frac{1}{\tau-1}\cos\left(2M+2\omega+2\Omega_e\right)+\frac{17}{2-\tau}\cos\left(4M+2\omega+2\Omega_e\right)\right)\right] \\
&+(1-\cos i)^2\left[\frac{1}{2+4\tau}\left(\cos\left(M+2\omega-2\Omega_e\right)\right)+\frac{7}{6+4\tau}\cos\left(3M+2\omega-2\Omega_e\right)\right. \\
&\left.+\frac{e}{2}\left(-\frac{1}{1+\tau}\cos\left(2M+2\omega-2\Omega_e\right)+\frac{17}{2+\tau}\cos\left(4M+2\omega-2\Omega_e\right)\right)\right] \\
&+2\sin^2 i\left[\frac{3}{2-4\tau}\cos\left(M+2\Omega_e\right)+\frac{3}{2+4\tau}\cos\left(M-2\Omega_e\right)\right. \\
&\left.\left.+\frac{9e}{4}\left(\frac{1}{1-\tau}\cos\left(2M+2\Omega_e\right)+\frac{1}{1+\tau}\cos\left(2M-2\Omega_e\right)\right)\right]\right\}
\end{aligned}
$$

扇谐与田谐

$$
\begin{aligned}
i_{short,22} = &-\frac{3}{4}J_{22}\left(\frac{r_e}{a}\right)^2 \sin i \\
&\times \left\{(1+\cos i)\left[\frac{e}{1-2\tau}\cos(M+2\omega+2\Omega_e) - \frac{7e}{3-2\tau}\cos(3M+2\omega-2\Omega_e)\right.\right. \\
&\left. -\frac{1}{1-\tau}\cos(2M+2\omega+2\Omega_e)\right] - (1-\cos i)\left[\frac{e}{1+2\tau}\cos(M+2\omega-2\Omega_e)\right. \\
&\left. -\frac{7e}{3+2\tau}\cos(3M+2\omega-2\Omega_e) - \frac{1}{1+\tau}\cos(2M+2\omega-2\Omega_e)\right] \\
&\left. -6e\left[\frac{1}{1-2\tau}\cos(M+2\Omega_e) - \frac{1}{1+2\tau}\cos(M-2\Omega_e)\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
\Omega_{short,22} = &-\frac{3}{4}J_{22}\left(\frac{r_e}{a}\right)^2 \\
&\times \left\{(1+\cos i)\left[\frac{e}{1-2\tau}\sin(M+2\omega+2\Omega_e) - \frac{7e}{3-2\tau}\sin(3M+2\omega+2\Omega_e)\right.\right. \\
&\left. -\frac{1}{1-\tau}\sin(2M+2\omega+2\Omega_e)\right] - (1-\cos i)\left[\frac{e}{1+2\tau}\sin(M+2\omega-2\Omega_e)\right. \\
&\left. -\frac{7e}{3+2\tau}\sin(3M+2\omega-2\Omega_e) - \frac{1}{1+\tau}\sin(2M+2\omega-2\Omega_e)\right] \\
&\left. +6e\cos i\left[\frac{1}{1-2\tau}\sin(M+2\Omega_e) + \frac{1}{1+2\tau}\sin(M-2\Omega_e)\right]\right\}
\end{aligned}
$$

扇谐与田谐

$$
\begin{aligned}
\omega_{short,22} = & -\frac{3}{4}\frac{J_{22}}{e}\left(\frac{r_e}{a}\right)^2 \\
& \times \left\{ -(1+\cos i)^2 \left[\frac{1}{2-4\tau}\sin(M+2\omega+2\Omega_e) - \frac{7}{6-4\tau}\sin(3M+2\omega+2\Omega_e) \right.\right. \\
& \left. + \frac{5e}{2-2\tau}\sin(2M+2\omega+2\Omega_e) - \frac{17e}{4-2\tau}\sin(4M+2\omega+2\Omega_e) \right] \\
& - (1-\cos i)^2 \left[\frac{1}{2+4\tau}\sin(M+2\omega-2\Omega_e) - \frac{7}{6+4\tau}\sin(3M+2\omega-2\Omega_e) \right. \\
& \left. + \frac{5e}{2+2\tau}\sin(2M+2\omega-2\Omega_e) - \frac{17e}{4+2\tau}\sin(4M+2\omega-2\Omega_e) \right] \\
& + \frac{9}{2}e\sin^2 i \left[\frac{1}{1-\tau}\sin(2M+2\Omega_e) + \frac{1}{1+\tau}\sin(2M-2\Omega_e) \right] \\
& \left. + 2\sin^2 i \left[\frac{3}{2-4\tau}\sin(M+2\Omega_e) + \frac{3}{2+4\tau}\sin(M-2\Omega_e) \right] \right\} \\
& - \Omega_{short,22}\cos i
\end{aligned}
$$

扇谐与田谐

$$
\begin{aligned}
M_{short,22} = &-\frac{9}{4}J_{22}\left(\frac{r_e}{a}\right)^2 \\
&\times\left\{(1+\cos i)^2\left[\frac{1}{1-\tau}\left(1-\frac{1}{2-2\tau}\right)\sin(2M+2\omega+2\Omega_e)\right.\right. \\
&-\frac{e}{1-2\tau}\left(1-\frac{1}{2-4\tau}\right)\sin(M+2\omega+2\Omega_e) \\
&\left.-\frac{7e}{3-2\tau}\left(1-\frac{3}{6-4\tau}\right)\sin(3M+2\omega+2\Omega_e)\right] \\
&-(1-\cos i)^2\left[\frac{e}{1+2\tau}\left(1-\frac{1}{2+4\tau}\right)\sin(M+2\omega-2\Omega_e)\right. \\
&-\frac{7e}{3+2\tau}\left(1-\frac{3}{6+4\tau}\right)\sin(3M+2\omega-2\Omega_e) \\
&\left.-\frac{1}{1+\tau}\left(1-\frac{1}{2+2\tau}\right)\sin(2M+2\omega-2\Omega_e)\right] \\
&+6e\sin^2 i\left[\frac{1}{1-2\tau}\left(1-\frac{1}{2-4\tau}\right)\sin(M+2\Omega_e)\right. \\
&\left.\left.+\frac{1}{1+2\tau}\left(1-\frac{1}{1+2\tau}\right)\sin(M-2\Omega_e)\right]\right\} \\
&-\omega_{short,22}-\Omega_{short,22}\cos i
\end{aligned}
$$

DSST半解析轨道计算软件部分（主要）配置参数

```
# body.frame                  = CIO/2003-based ITRF accurate EOP
body.frame                    = CIO/2010-based ITRF simple EOP
# body.frame                  = CIO/2010-based ITRF accurate EOP
```

```
## date of the orbital parameters (UTC)
orbit.date = 2011-12-12T11:57:20.000

### Keplerian elements
## Semi-major axis (km)
orbit.keplerian.a = 7204.535848109436
## Eccentricity
orbit.keplerian.e = 0.0012402238462686
## Inclination (degrees)
orbit.keplerian.i = 98.74341600466740
## Right Ascension of Ascending Node (degrees)
orbit.keplerian.raan = 43.32990110790340
## Perigee Argument (degrees)
orbit.keplerian.pa = 111.1990175076630
## Anomaly (degrees)
```

```
central.body.rotation.rate = 7.292115e-5
## Central body gravity potential degree
central.body.degree = 6
## Central body gravity potential order
central.body.order  = 6
## short period limits
max.degree.zonal.short.periods                            = 6

# typically min(max.degree.zonal.short.periods - 1, 4)
max.eccentricity.power.zonal.short.periods                = 4

# typically 2 * max.degree.zonal.short.periods + 1
max.frequency.true.longitude.zonal.short.periods          = 13

max.degree.tesseral.short.periods                         = 6
max.order.tesseral.short.periods                          = 6

# typically min(max.degree.tesseral.short.periods - 1, 4)
max.eccentricity.power.tesseral.short.periods             = 4

# typically min(max.degree.tesseral.short.periods + max.eccentri
max.frequency.mean.longitude.tesseral.short.periods       = 10

max.degree.tesseral.m.dailies.short.periods               = 6
max.order.tesseral.m.dailies.short.periods                = 6

# typically min(max.degree.tesseral.short.periods - 2, 4)
max.eccentricity.power.tesseral.m.dailies.short.periods = 4
```

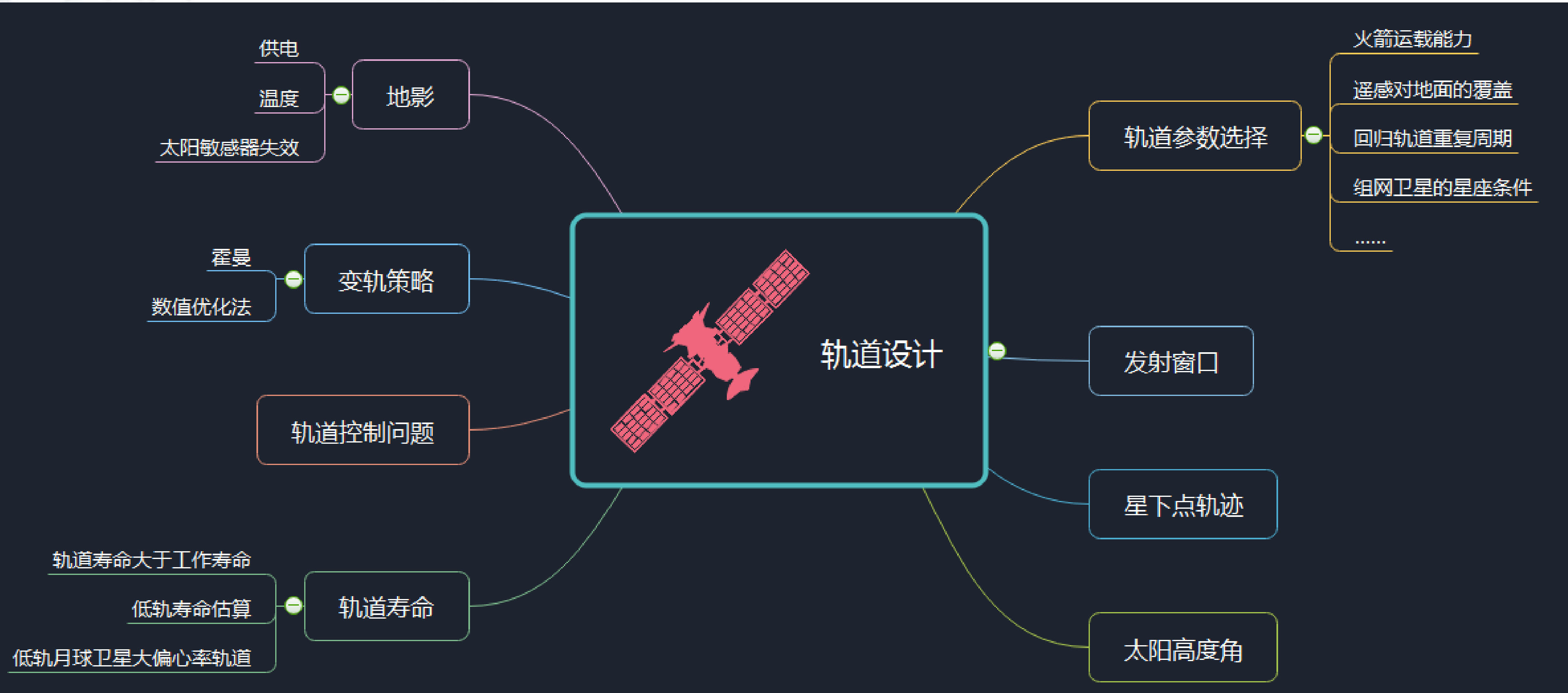
DSST半解析轨道计算软件部分（主要）配置参数

```
## Duration (days)
duration.in.days = 365.0
## Time step between printed elements (secon
output.step = 86400.0

### Output parameters
## Output Keplerian parameters in output fi
output.Keplerian = true
## Output equinoctial parameters in output f
output.equinoctial = true
## Output Cartesian parameters in output fi
output.Cartesian = true
## Short period coefficients to output (comm
## in order to get the list of available coe
## run with a value set to all and then look
## (a), (h), (k), (p), (q) and (L) parts do
## describe the 6 columns that correspond to
## Beware that the list is huge ...
output.short.period.coefficients = none
```

```
## seconds, and a position tolerance in meters (beware, i
## really the global error, it correspond to a local esti
## at each step, often difficult to relate to final globa
min.variable.integration.step                  =  6000.0
max.variable.integration.step                  = 86400.0
position.tolerance.variable.integration.step =     1.0

## interpolation grid specification
## only one of the two following options can be set: eith
## of points or maximum gap. If both options are specifie
## will be generated.
## if neither option is set, a fixed number of 3 points w
# fixed number of interpolation points at each mean eleme
# fixed.number.of.interpolation.points = 3
# maximum gap between interpolation points
max.time.gap.between.interpolation.points = 8640
```

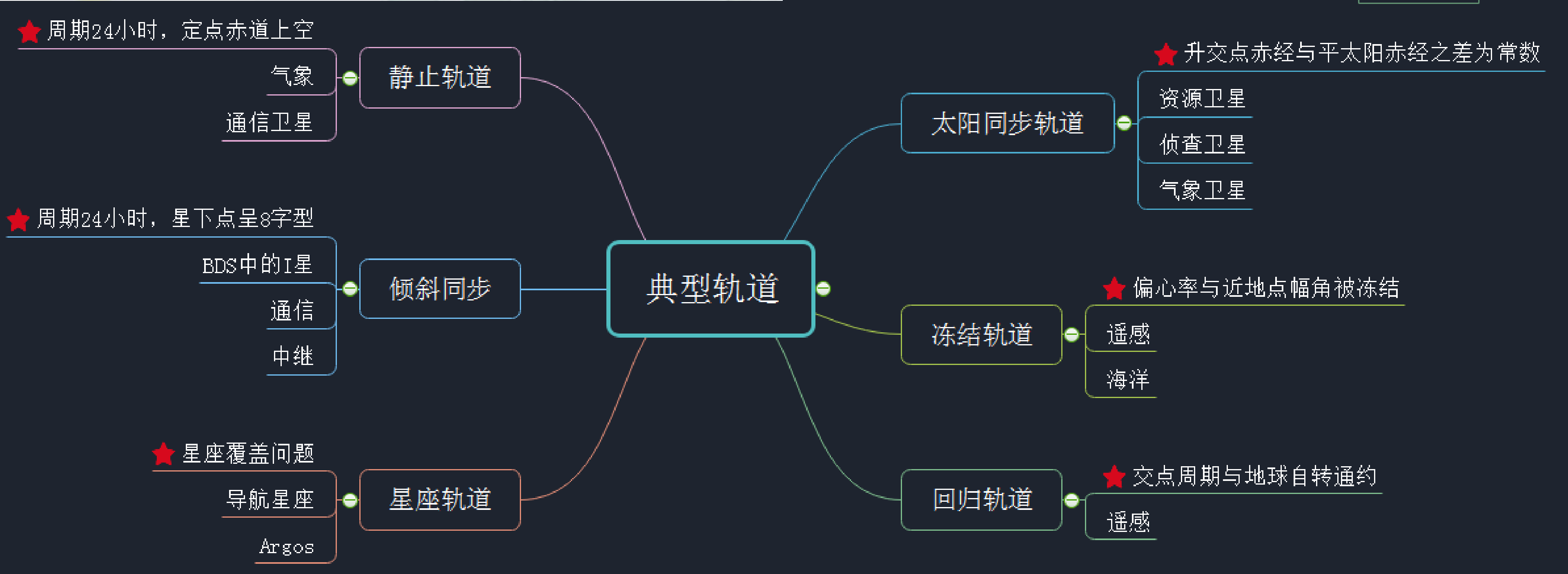
摄动法应用及轨道设计问题

轨道设计和卫星使命明密切相关，不同用途的卫星，很难统一归纳轨道设计内容。下图是一些常见的地球卫星轨道设计共性问题。



轨道设计步骤与常用地球卫星轨道

- ➤问题提出
- ➤模型建立
- ➤数学求解
- ➤结果分析




Q&A!