中国科学院上海天文台
Shanghai Astronomical Observatory, Chinese Academy of Science

中国科学院大学
University of Chinese Academy of Sciences

轨道测量与误差修正

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2019年秋季

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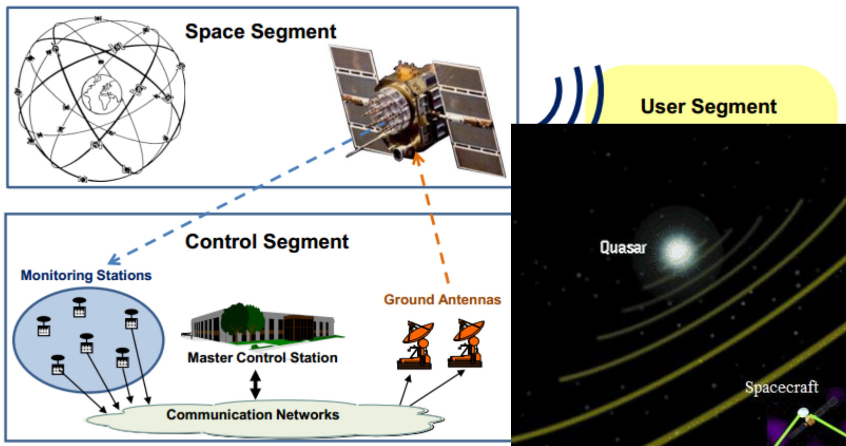
主要内容

- 轨道测量概述
- 电磁波传播
- 对电磁波有影响的空间环境
- 信号传播介质改正
- 台站坐标潮汐改正
- 相对论改正等其他误差
- 偏导数问题

航天测控系统

航天测控系统
监控和显示系统
时统系统
地地通信系统
天地通信与数据传输
数据处理系统
轨道确定和预报
姿态确定
数据分析
......
跟踪测轨系统
光学
无线电
遥测系统
工程参数
航天员生理参数
天地回路控制数据
载荷参数
遥控系统
轨道控制
姿态控制
数据管理
......
辅助支持

测量跟踪技术



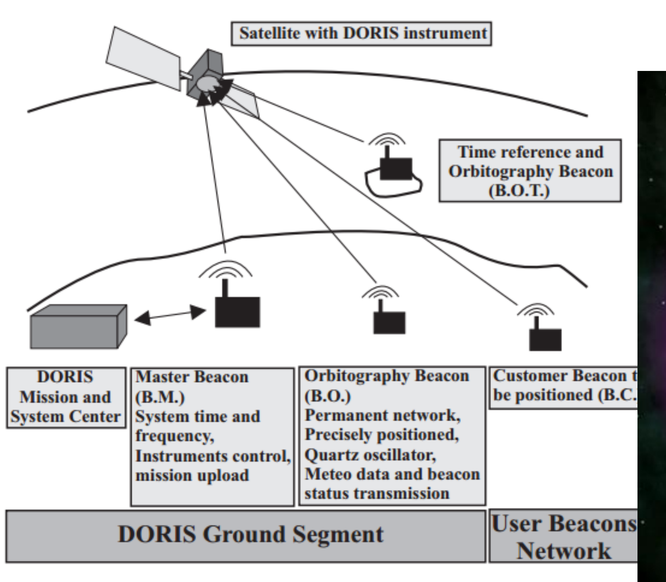
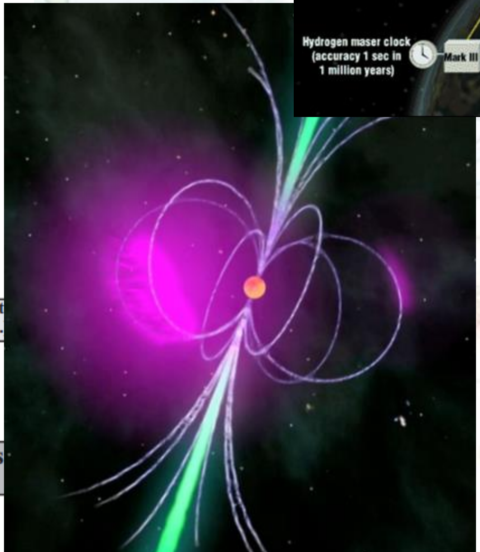
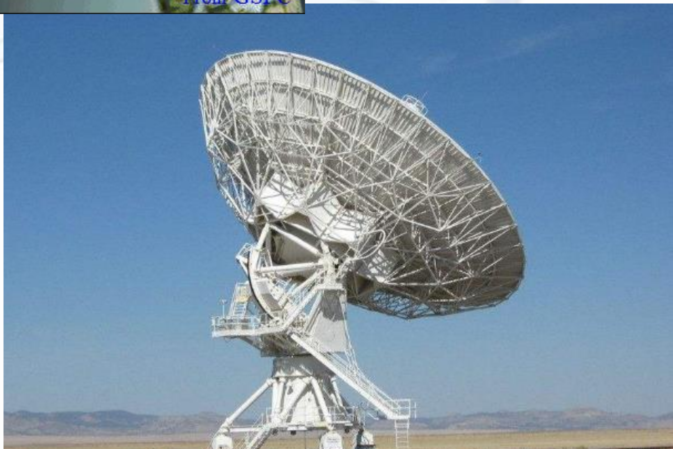
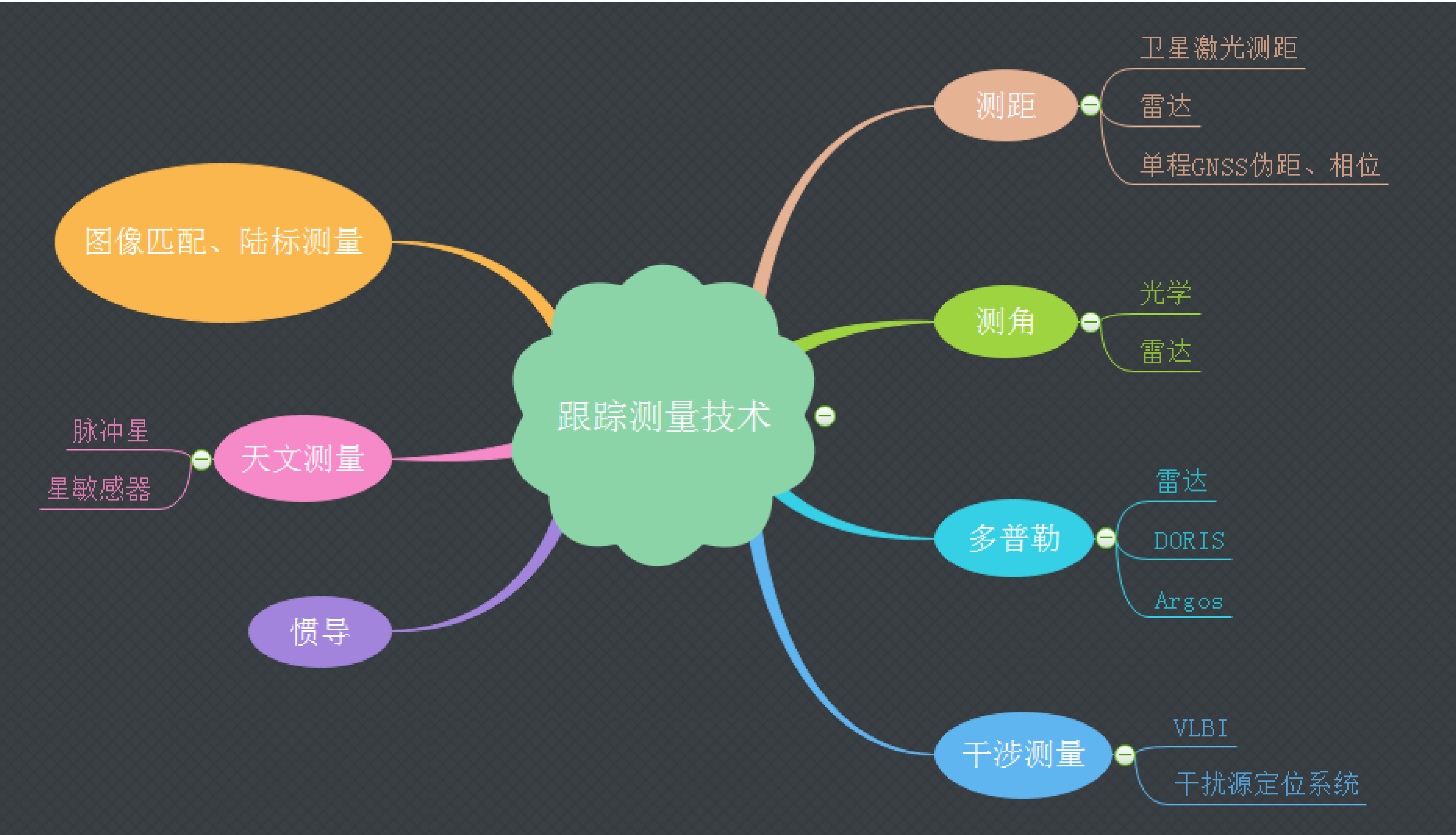


Figure 6.21 DORIS System Overview





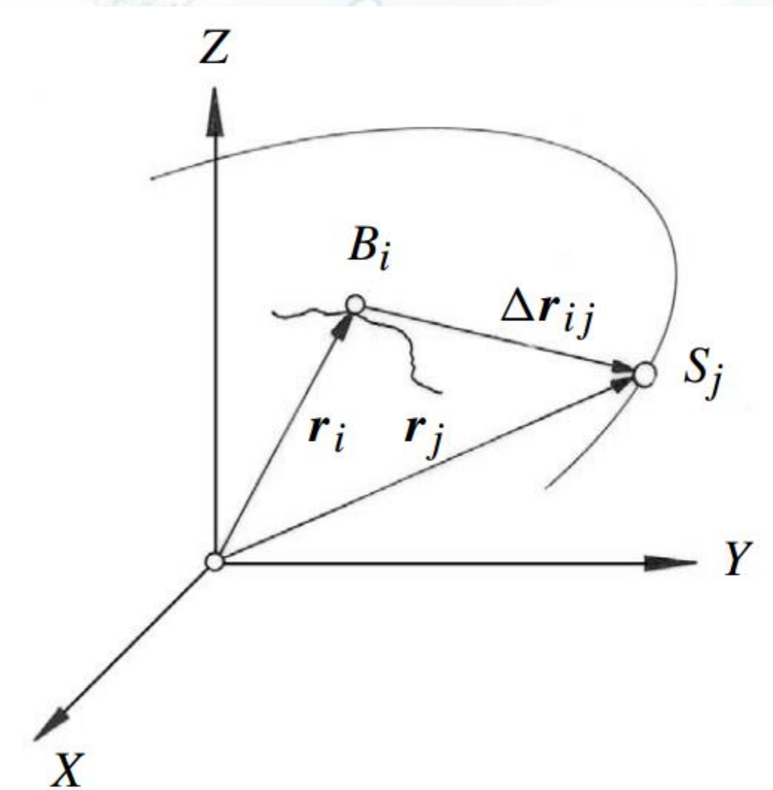
测量跟踪技术

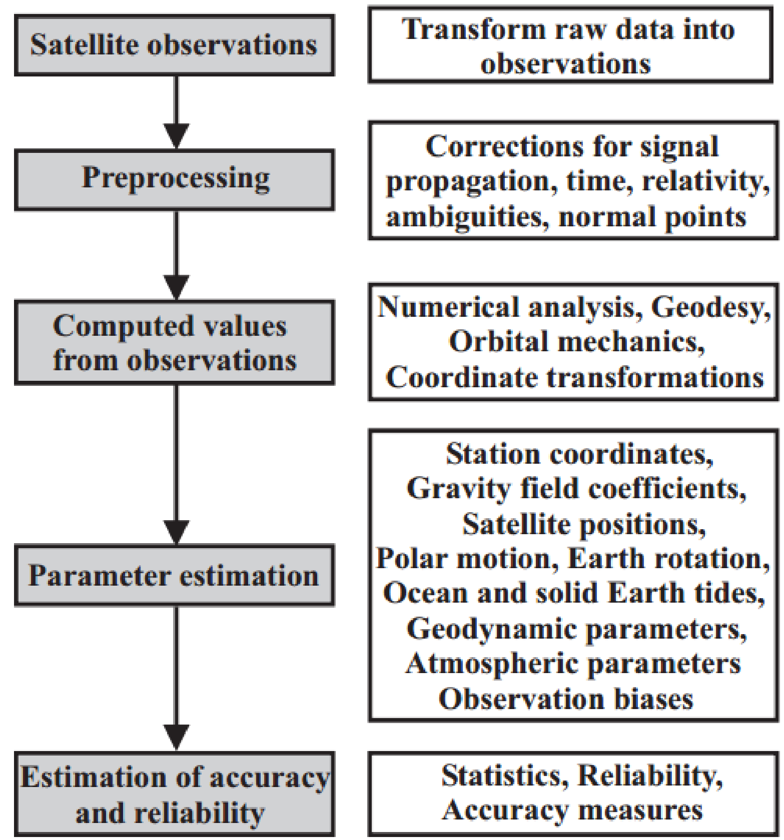


跟踪测量技术
测距
卫星激光测距
雷达
单程GNSS伪距、相位
测角
光学
雷达
多普勒
雷达
DORIS
Argos
干涉测量
VLBI
干扰源定位系统
图像匹配、陆标测量
天文测量
脉冲星
星敏感器
惯导

Satellite Geodesy as a Parameter Estimation Problem

$$\boldsymbol{r}_j(t) = \boldsymbol{r}_i(t) + \Delta\boldsymbol{r}_{ij}(t)$$

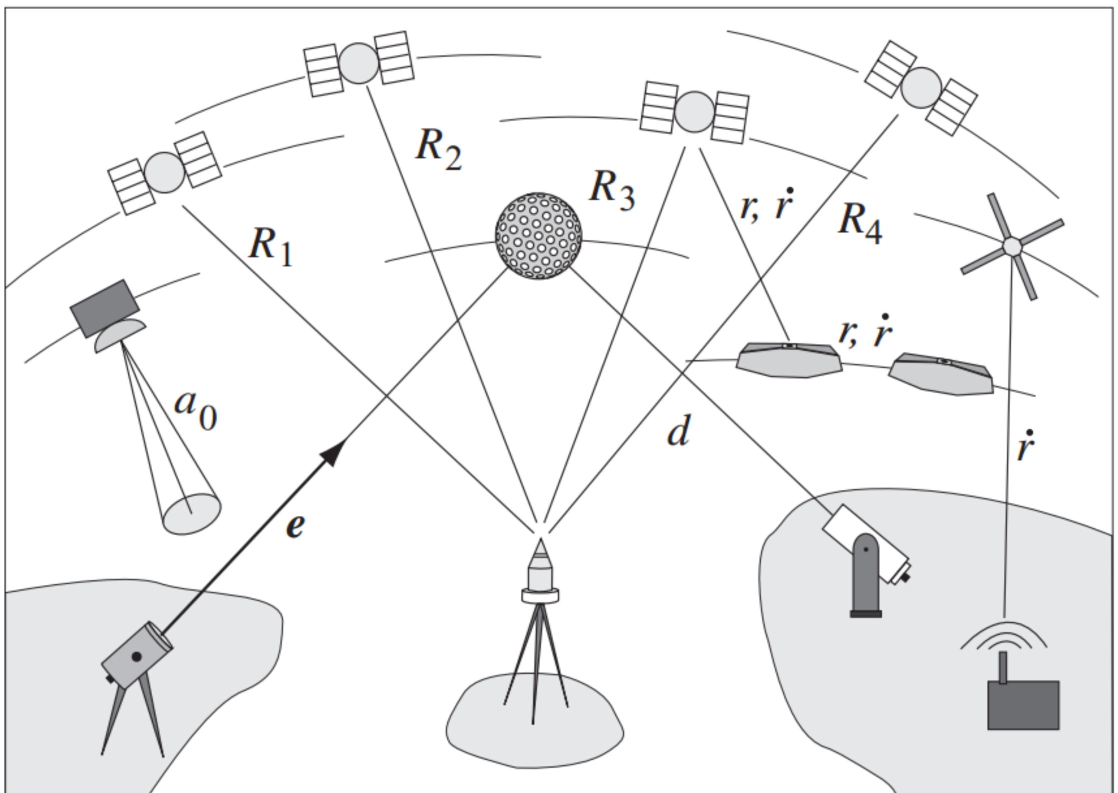




Functional scheme for the use of satellite observations

按平台分类

- Earth based techniques (ground station → satellite),
- satellite based techniques (satellite → ground station),
- inter-satellite techniques (satellite → satellite).



电磁波传播

$$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t}$$

$$\nabla \times \boldsymbol{H} = \varepsilon \frac{\partial \boldsymbol{E}}{\partial t} ,$$

$$\nabla \cdot \boldsymbol{E} = 0 ,$$

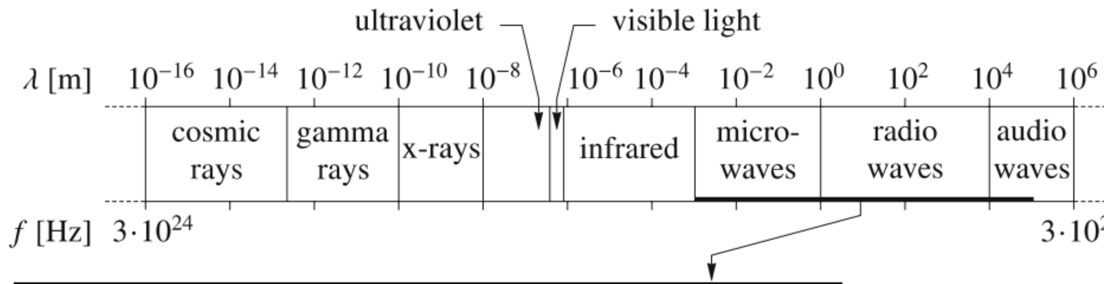
$$\nabla \cdot \boldsymbol{H} = 0 .$$

$$\nabla \times (\nabla \times \boldsymbol{E}) = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E} ,$$

$$= -\mu \frac{\partial}{\partial t} \nabla \times \boldsymbol{H} ,$$

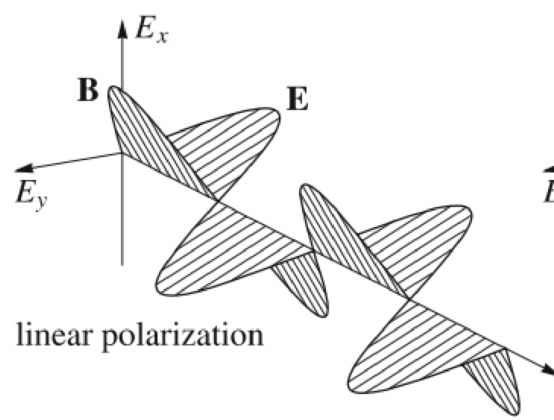
$$\nabla^2 \mathbf{E} = \varepsilon\mu \frac{\partial^2}{\partial t^2} \mathbf{E}$$

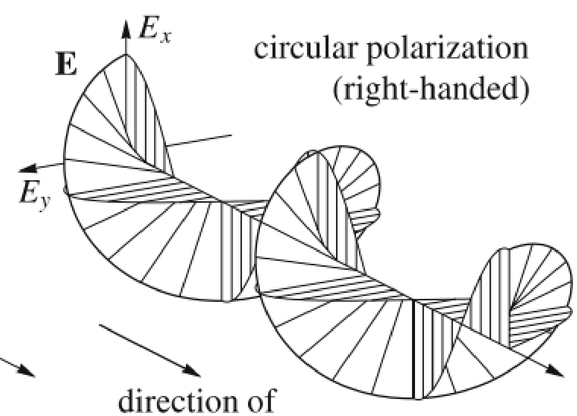
$$\nabla^2 \mathbf{H} = \varepsilon\mu \frac{\partial^2}{\partial t^2} \mathbf{H}$$



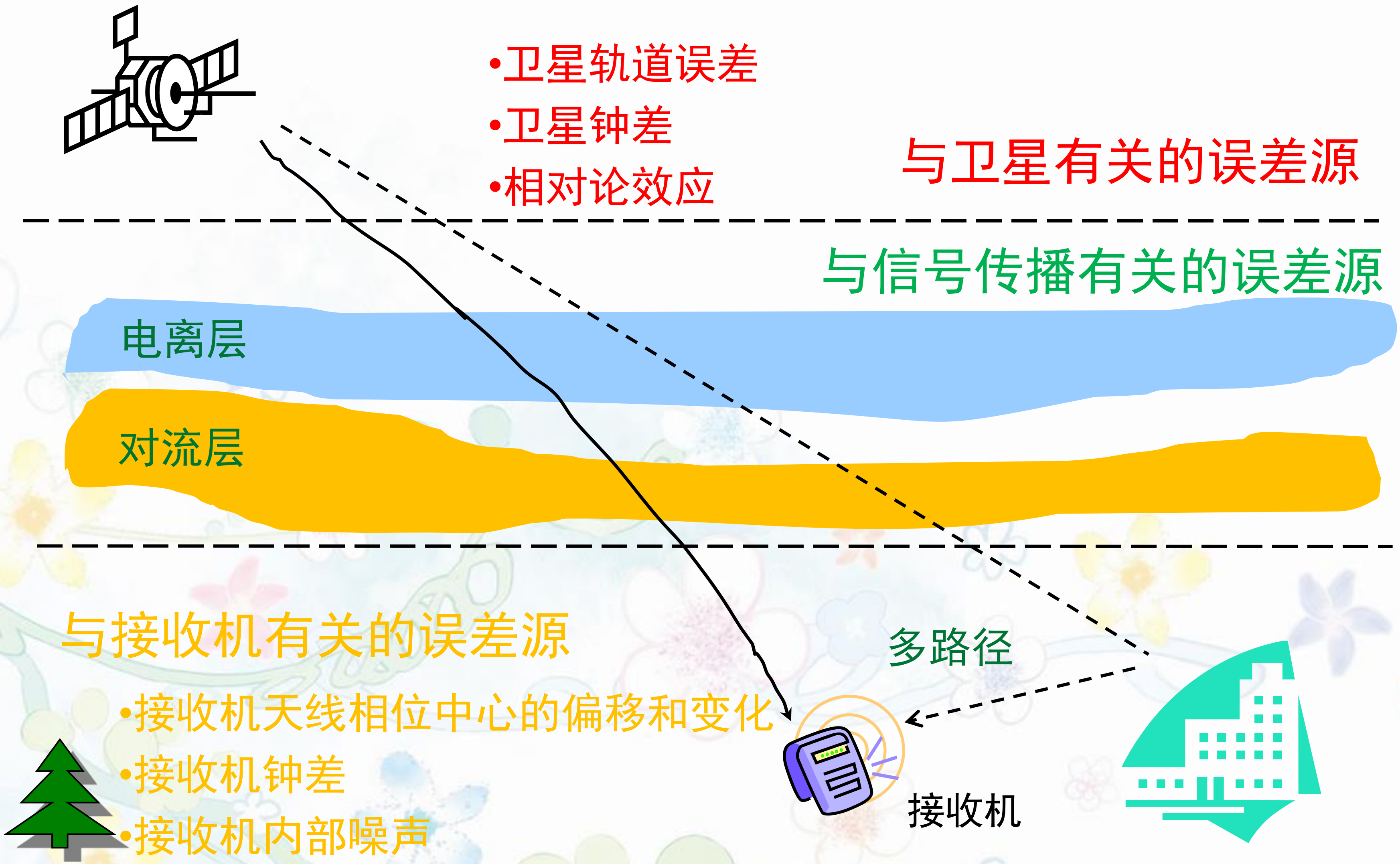
Notation	Wavelength λ	Frequency f
Extremely high frequency (EHF)	0.1–1 cm	300–30 GHz
Super high frequency (SHF)	1–10 cm	30–3 GHz
Ultra high frequency (UHF)	10–100 cm	3–0.3 GHz
Very high frequency (VHF)	1–10 m	300–30 MHz
High frequency (HF)	10–100 m	30–3 MHz
Medium frequency (MF)	0.1–1 km	3–0.3 MHz
Low frequency (LF)	1–10 km	300–30 kHz
Very low frequency (VLF)	10–100 km	30–3 kHz

Band	f [GHz]
K	26.5–18
Ku	18–12.4
X	12.4–8
C	8–4
S	4–2
L	2–1



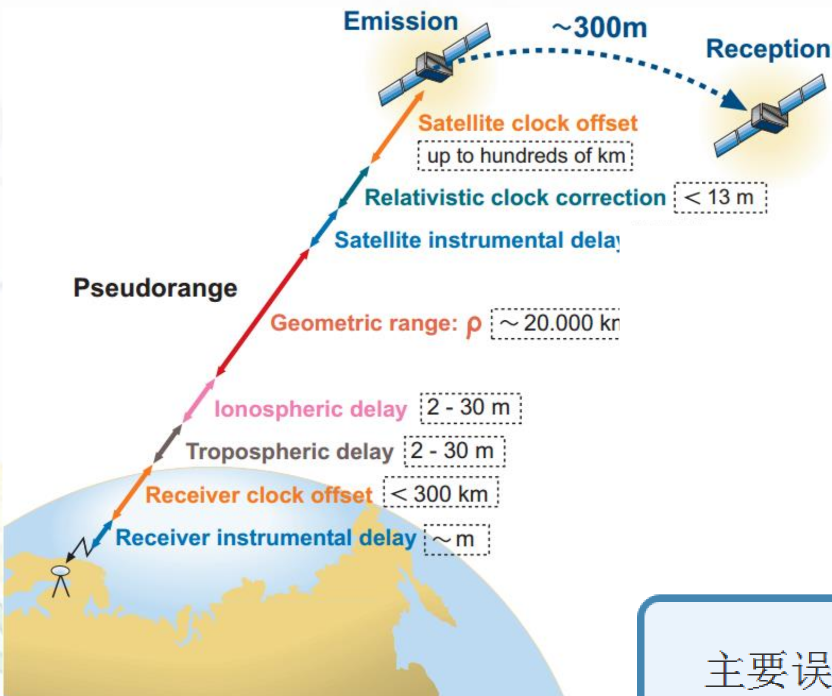


主要的误差源类型

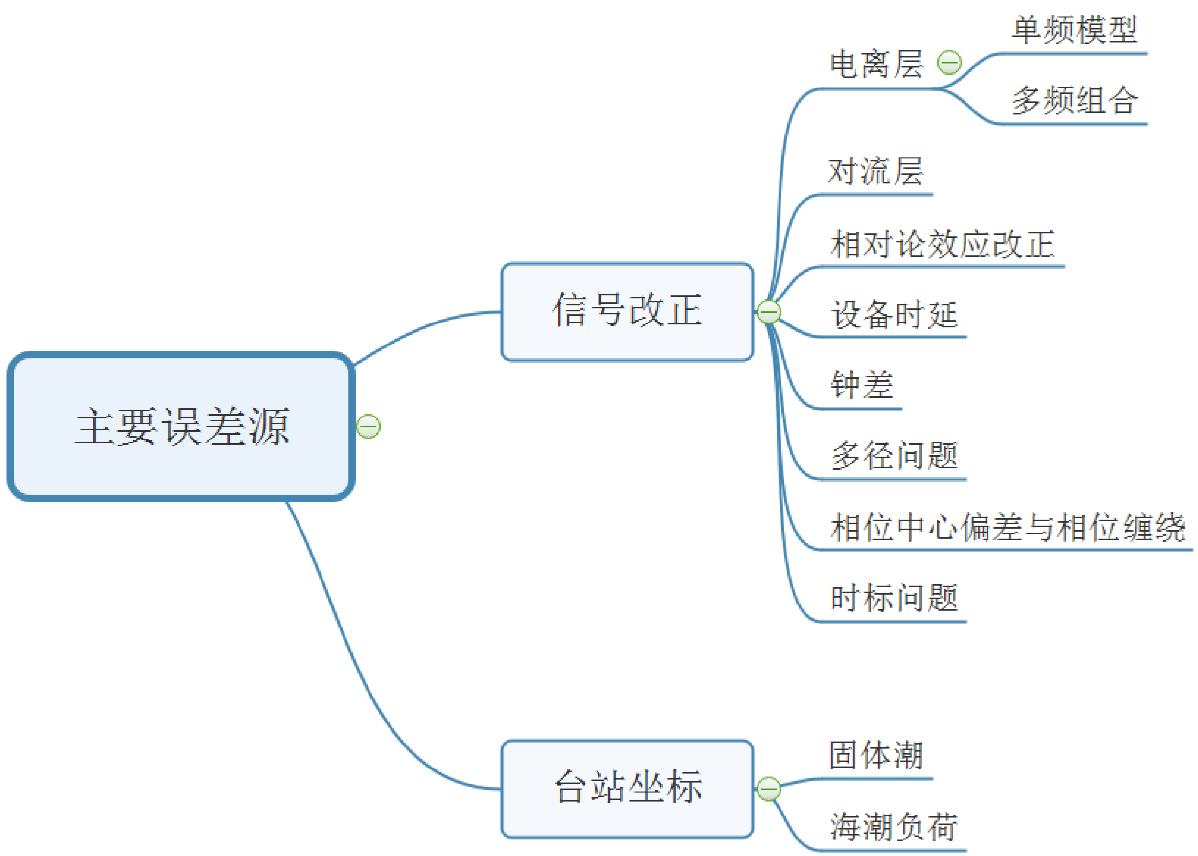


•卫星轨道误差
•卫星钟差
•相对论效应
与卫星有关的误差源
与信号传播有关的误差源
电离层
对流层
与接收机有关的误差源
•接收机天线相位中心的偏移和变化
•接收机钟差
•接收机内部噪声
多路径
接收机

主要误差源量级

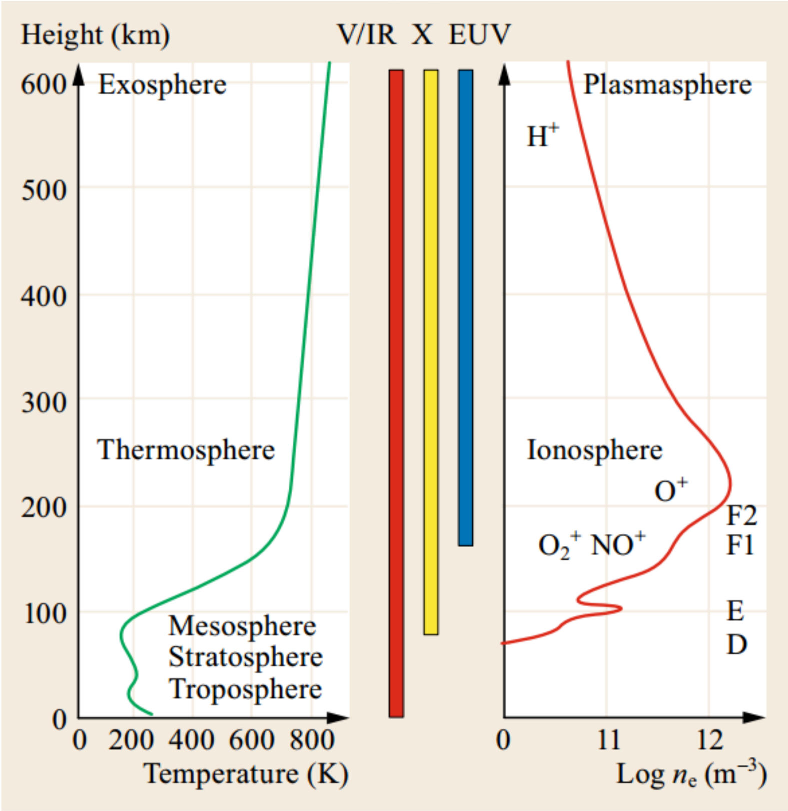


Emission
~300m
Reception
Satellite clock offset
up to hundreds of km
Relativistic clock correction
< 13 m
Satellite instrumental dela
Pseudorange
Geometric range: ρ
~20.000 kr
Ionospheric delay
2 - 30 m
Tropospheric delay
2 - 30 m
Receiver clock offset
< 300 km
Receiver instrumental delay
~m



主要误差源
信号改正
电离层
单频模型
多频组合
对流层
相对论效应改正
设备时延
钟差
多径问题
相位中心偏差与相位缠绕
时标问题
台站坐标
固体潮
海潮负荷

空间环境之电离层



Vertical structure of the electron density of the ionosphere (*right*) in comparison with the neutral atmosphere temperature (*left*) and solar radiation penetration depths (*middle*)

$$\mathrm{VTEC} = \int n_e \mathrm{d}h$$

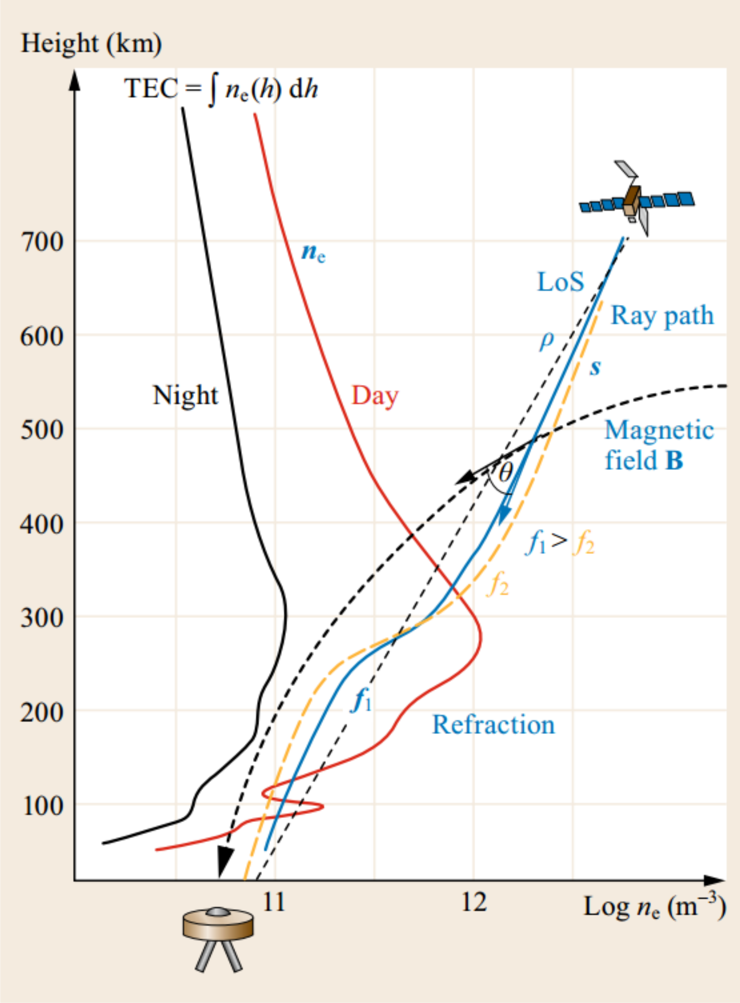
$$\mathrm{STEC} = \int n_e \mathrm{d}s$$

$$n_p = 1 + \frac{c_2}{f^2} + \frac{c_3}{f^3} + \frac{c_4}{f^4} \cdots$$

$$n_g = 1 - \frac{c_2}{f^2} - \frac{2c_3}{f^3} - \frac{3c_4}{f^4} \cdots$$

$$\Delta S_{iono,p} = -\frac{40.3}{f^2}\int_{SV}^{User} n_e\, dl \qquad \Delta S_{iono,g} = \frac{40.3}{f^2}\int_{SV}^{User} n_e\, dl$$

双频处理



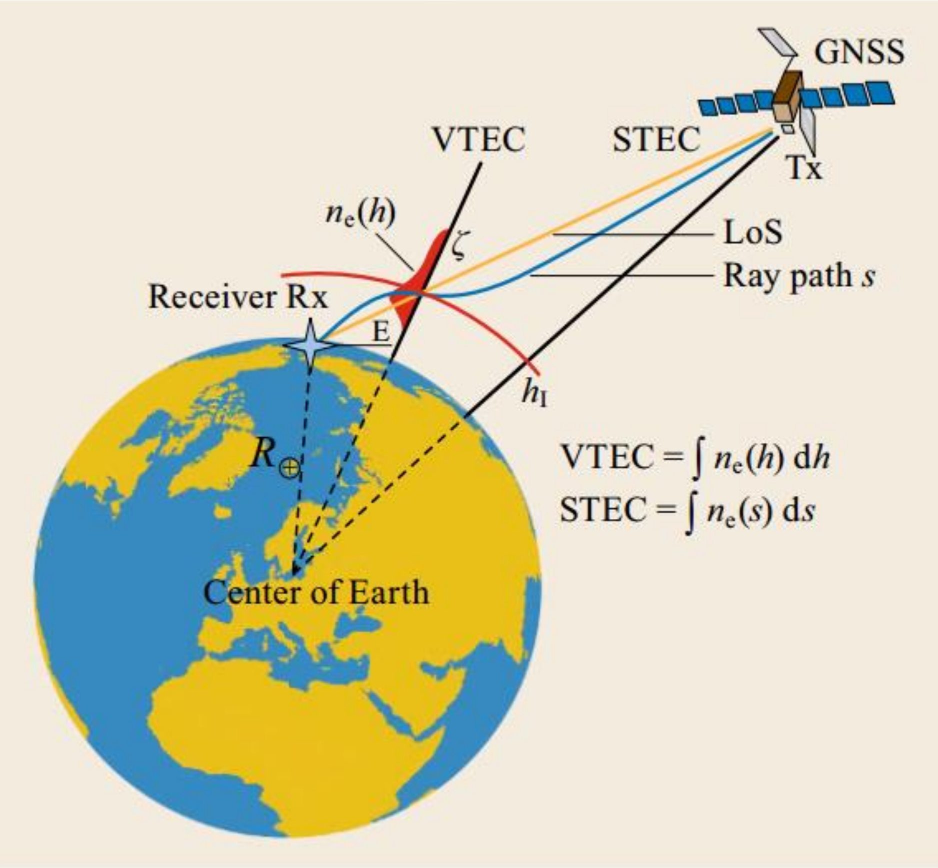
Scheme of transionospheric radio wave propagation at two frequencies f_1 and f_2 in the presence of the geomagnetic field B

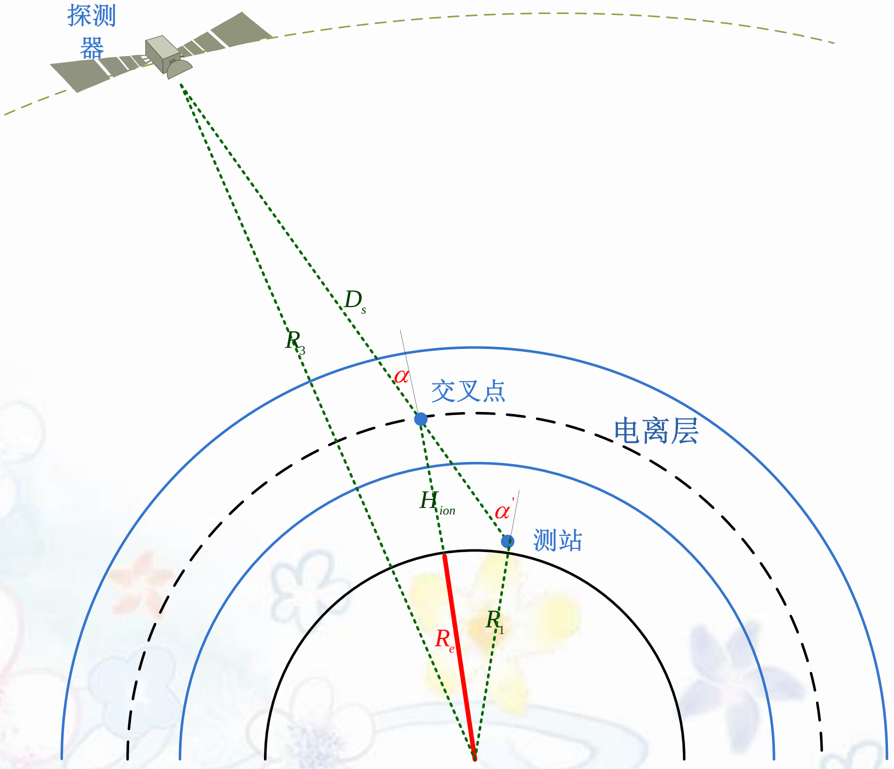
$$\rho_c = \gamma_1 \rho_1 - \gamma_2 \rho_2$$

$$\gamma_1 = \frac{f_1^2}{f_1^2 - f_2^2}$$

$$\gamma_2 = \frac{f_2^2}{f_1^2 - f_2^2}.$$

映射函数





$$F=\frac{1}{\cos(\alpha)}=\frac{1}{\sqrt{1-\left(\frac{Re}{Re+H_{ion}}\sin(\alpha')\right)^2}}$$

GPS中的K模型

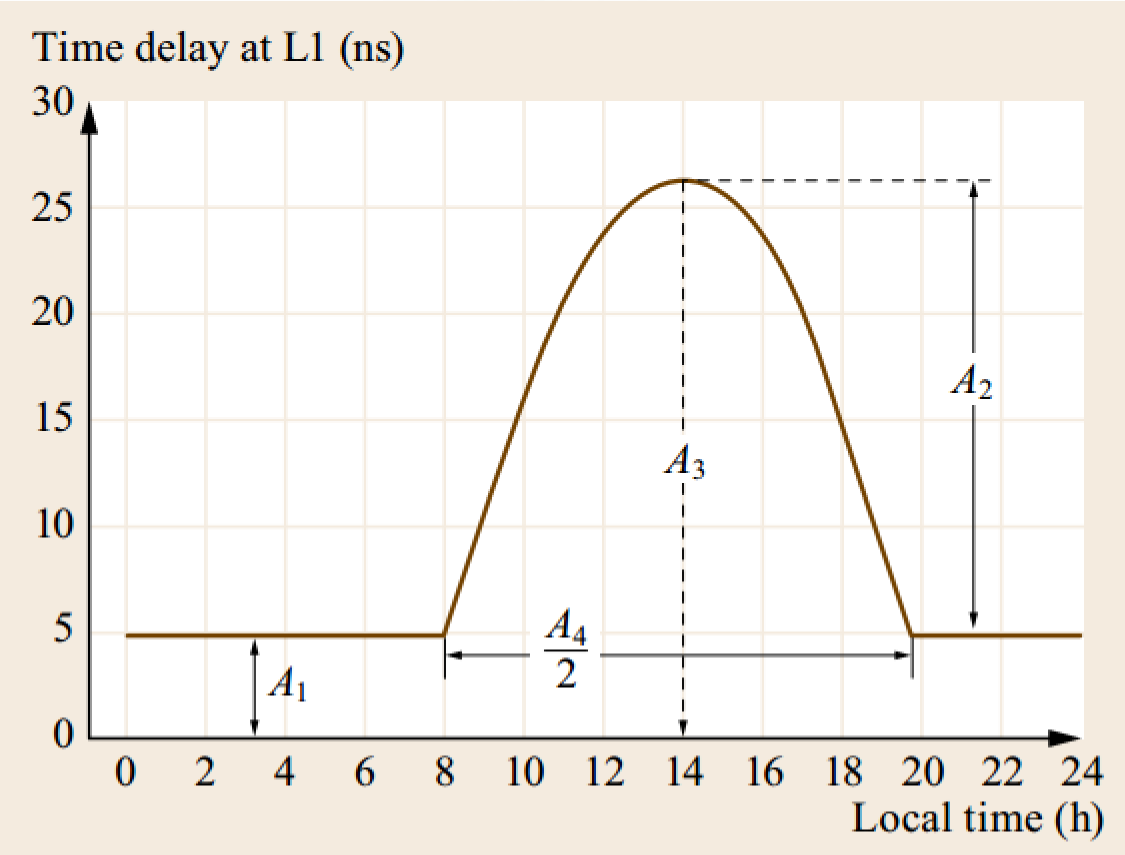


Illustration of the Klobuchar GPS correction model

$$T_{\text{ion}} = A_1 + A_2 \cos\left[\frac{2\pi(t_{\text{GPS}} - A_3)}{A_4}\right]$$

$$A_2 = \sum_{n=0}^{3} \alpha_n \phi_{\text{m}}$$

$$A_4 = \sum_{n=0}^{3} \beta_n \phi_{\text{m}}$$

BDS3基本导航电离层模型

$$T_{ion} = F \cdot K \cdot \left[A_0 + \sum_{i=1}^{9} \alpha_i A_i \right]$$

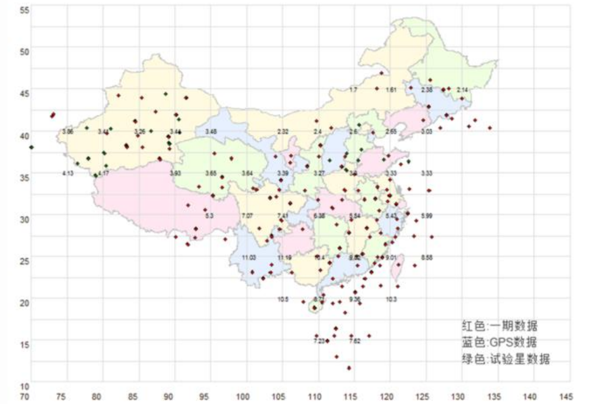
$$\begin{cases} \beta_j = \sum_{k=0}^{12} \left(a_{k,j} \cdot \cos \omega_k t_k + b_{k,j} \cdot \sin \omega_k t_k \right) \\ \omega_k = \dfrac{2\pi}{\mathrm{T}_k} \end{cases}$$

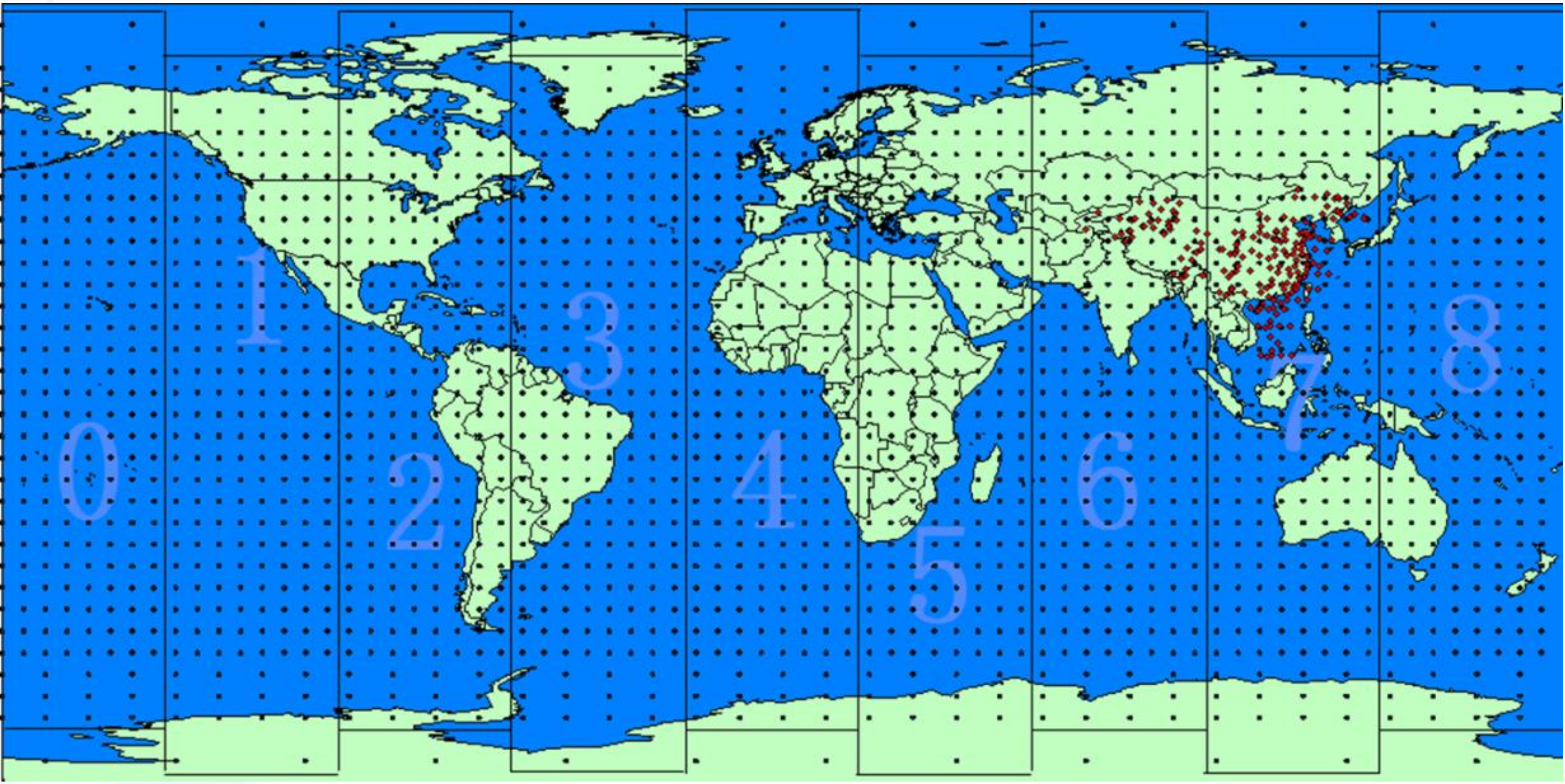
BDSSH 模型非发播系数预报周期表（单位：TECu）

参数 / 编号 k	编号 i	1	2	3	4	5
	n_i/m_i	3/0	3/1	3/-1	3/2	3/-2
0	$a_{k,i}$	-0.61	-1.31	-2.00	-0.03	0.15
1	$a_{k,i}$	-0.51	-0.43	0.34	-0.01	0.17
	$b_{k,i}$	0.23	-0.20	-0.31	0.16	-0.03
2	$a_{k,i}$	-0.06	-0.05	0.06	0.17	0.15
	$b_{k,i}$	0.02	-0.08	-0.06	-0.11	0.15
3	$a_{k,i}$	0.01	-0.03	0.01	-0.01	0.05
	$b_{k,i}$	0	-0.02	-0.03	-0.05	-0.01
4	$a_{k,i}$	-0.01	0	0.01	0	0.01
	$b_{k,i}$	0	-0.02	0.01	0	-0.01
5	$a_{k,i}$	0	0	0.03	0.01	0.02

$$\begin{cases} P_{n,n}(\sin\varphi') = (2n-1)!!(1-(\sin\varphi')^2)^{n/2} & n = m \\ P_{n,m}(\sin\varphi') = \sin\varphi' \cdot (2m+1) \cdot P_{m,m}(\sin\varphi') & n = m+1 \\ P_{n,m}(\sin\varphi') = ((2n-1)\sin\varphi' P_{n-1,m}(\sin\varphi') - (n+m-1)P_{n-2,m}(\sin\varphi')) \big/ (n-m) & \text{其他} \end{cases}$$

格网电离层


红色:一期数据
蓝色:GPS数据
绿色:试验星数据
55
50
45
40
35
30
25
20
15
10
70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 145


0
1
2
3
4
5
6
7
8

球谐模型问题

$$VTEC(\phi,\mathrm{s}) = \sum_{l=1}^{N}\sum_{m=1}^{l}\overline{P}_{lm}(\sin\phi)\cos(ms)\alpha_{lm} + \sum_{l=1}^{N}\sum_{m=1}^{l}\overline{P}_{lm}(\sin\phi)\sin(ms)\beta_{lm}$$

$$\sum_{l=1}^{N}\overline{P}_{l}(\sin\phi)\gamma_{l} + \zeta$$

$$\mathbf{A} = \left[\begin{matrix} [[A_{11}] \quad [A_{21} \quad A_{22}] \quad \cdots \quad [A_{l1} \quad \cdots \quad A_{ll}]] \quad [[[B_{11}] \quad [B_{21} \quad B_{22}] \quad \cdots \quad [B_{l1} \quad \cdots \quad B_{ll}]]] \quad [C_0 \quad C_1 \quad \cdots \quad C_n]] \end{matrix} \right.$$

[redacted]

$$\mathbf{X} = [[[\alpha_{11}] \quad [\alpha_{21} \quad \alpha_{22}] \quad \cdots \quad [\alpha_{l1} \quad \cdots \quad \alpha_{ll}]] \quad [[\beta_{11}] \quad [\beta_{21} \quad \beta_{22}] \quad \cdots \quad [\beta_{l1} \quad \cdots \quad \beta_{ll}]] \quad [\gamma_0 \quad \gamma_1 \quad \cdots \quad \gamma_n]]^T$$

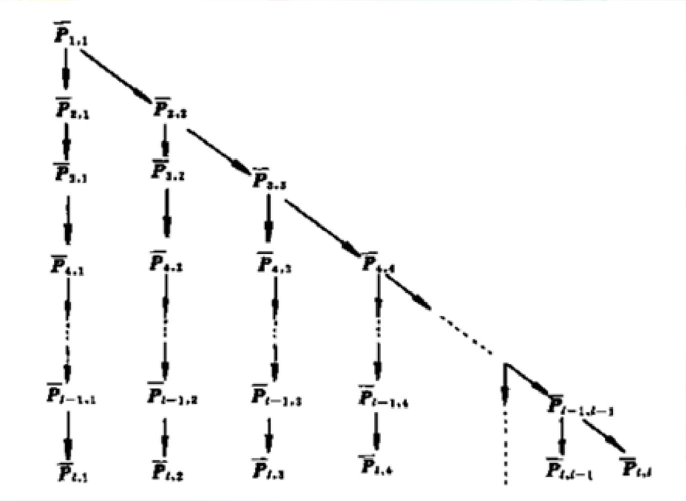
$$\begin{cases} A_{lm} = \overline{P}_{lm}(\sin\phi)\cos(ms) \\ B_{lm} = \overline{P}_{lm}(\sin\phi)\sin(ms) \\ C_{l} = \overline{P}_{l}(\sin\phi) \end{cases}$$

Legendre polynomials算法

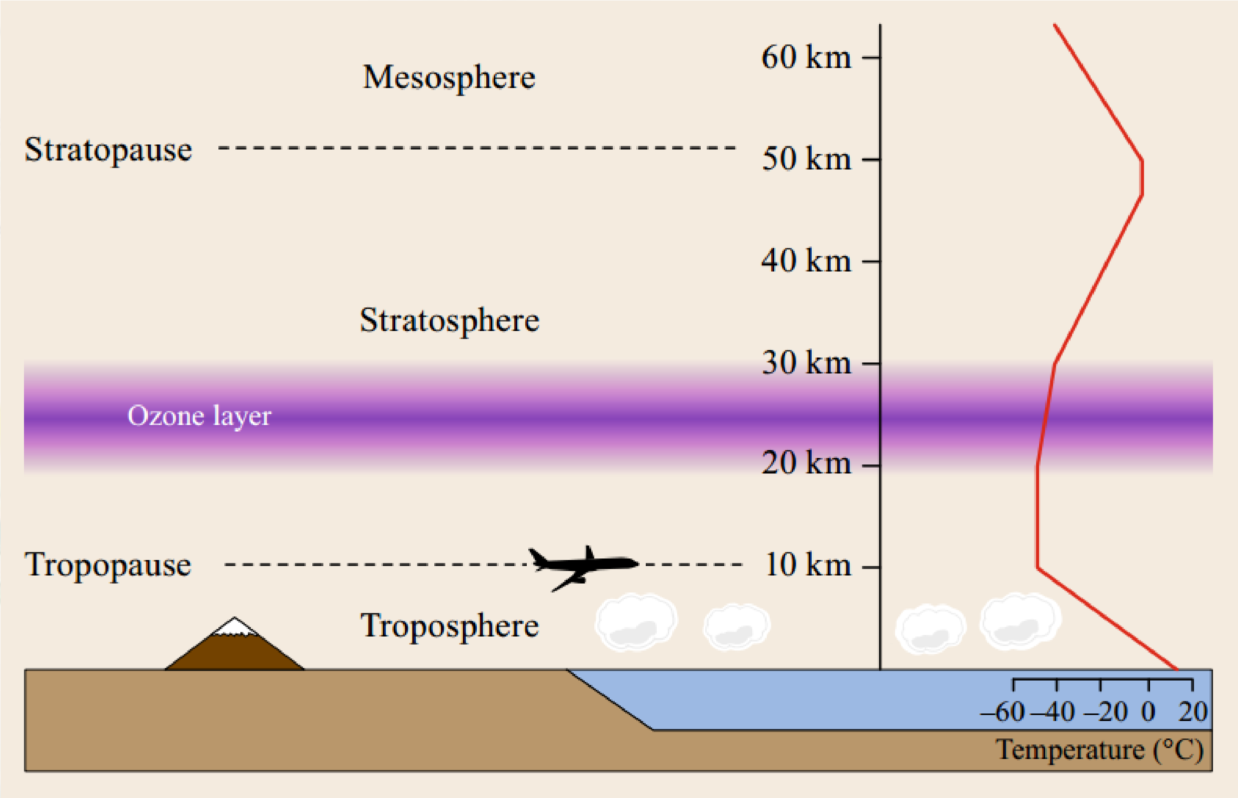
$$\overline{P}_l(u)=\sqrt{\frac{2l+1}{2l-1}}\left[\left(1-\frac{1}{l}\right)u\overline{P}_{l-1}(u)-\sqrt{\frac{2l-1}{2l-3}}\left(1-\frac{1}{l}\right)\overline{P}_{l-2}(u)\right],l\geq 2$$

$$\overline{P}_0(u)=1,\overline{P}_1(u)=\sqrt{3}u$$

$$\begin{cases}\overline{P}_{1,1}(u)=\sqrt{3\left(1-u^2\right)}\\ \overline{P}_{l,l}(u)=\sqrt{\frac{2l+1}{2l}}\sqrt{1-u^2}\,\overline{P}_{l-1,l-1}(u)\\ \overline{P}_{l,m}(u)=\sqrt{\frac{(2l+1)(2l-1)}{(l+m)(l-m)}}u\overline{P}_{l-1,m}(u)\\ \qquad -\sqrt{\frac{(2l+1)(l-1+m)(l-1-m)}{(2l-3)(l+m)(l-m)}}\overline{P}_{l-2,m}(u)\\ l\geq 2,m=1,2,\ldots,l-1\\ \overline{P}_{i,j}(u)=0,i<j\end{cases}$$



空间环境之对流层



$$\delta\rho = 10^{-6}\int N ds$$

$$N = (n-1)10^6$$

$$N = N_\mathrm{d} + N_\mathrm{w}$$

The dry component,or hydrostatic component, accounts for about 90% of the total effect .

$$\Delta\rho_t(El) = \tau_d m_d(El) + \tau_w m_w(El)$$

Marini and Murray模型

$$\Delta\rho_{RF} = \frac{f(\lambda)}{f(\phi, H)} \times \frac{A+B}{\sin\gamma + \frac{B/(A+B)}{\sin\gamma + 0.01}}$$

$$A = 0.002357P + 0.000141W_1$$

$$B = 1.084\times10^{-8} \times P \times T \times K + \frac{2\times4.734\times10^{-8}\times P^2}{T\times(3-\frac{1}{K})}$$

$$f(\lambda) = 0.9650 + \frac{0.0164}{\lambda^2} + \frac{0.000228}{\lambda^4}$$

$$W_1 = \frac{W}{100}\times6.11\times10^{\frac{7.5\times(T-273.15)}{237.3+(T-273.15)}}$$

$$f(\phi, H) = 1 - 0.0026\cos 2\phi - 3.1\times10$$

$$K = 1.163 - 0.00968\cos 2\phi - 0.00104T + 0.00001435P$$

MIT映射函数

$$m(El) = \frac{1 + \frac{a}{1+\frac{b}{1+c}}}{\sin(El) + \frac{a}{\sin(El)+\frac{b}{\sin(El)+\frac{b}{\sin(El)+c}}}}$$

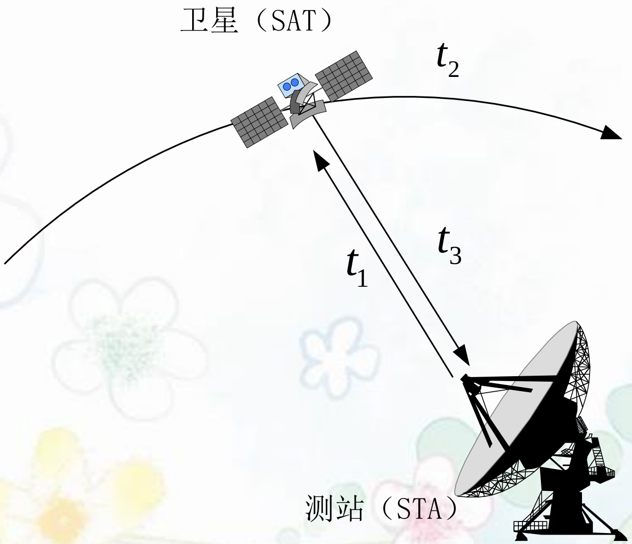
干模型

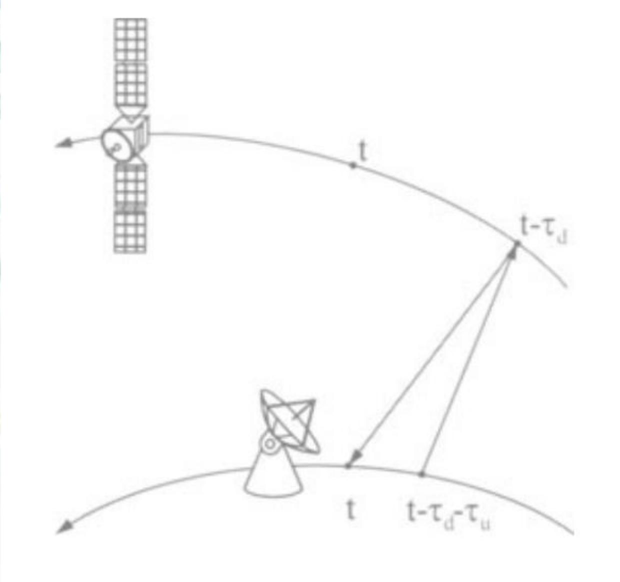
$$\begin{aligned} a &= [1.2320 + 0.0130\cos\phi - 0.0209H_s \\ &\quad +0.00215(T_s - T_0)] \times 10^{-3} \\ b &= [3.1612 - 0.1600\cos\phi - 0.0331H_s \\ &\quad +0.00206(T_s - T_0)] \times 10^{-3} \\ c &= [71.244 - 4.293\cos\phi - 0.149H_s \\ &\quad -0.0021(T_s - T_0)] \times 10^{-3} \end{aligned}$$

湿模型

$$\begin{aligned} a &= [0.583 - 0.011\cos\phi - 0.052H_s \\ &\quad +0.0014(T_s - T_0)] \times 10^{-3} \\ b &= [1.402 + 0.102\cos\phi - 0.101H_s \\ &\quad +0.0020(T_s - T_0)] \times 10^{-3} \\ c &= [45.85 - 1.91\cos\phi - 1.29H_s \\ &\quad +0.015(T_s - T_0)] \times 10^{-3} \end{aligned}$$

双程测距与光行时





$$\rho_t = \frac{\rho_d + \rho_u}{2}$$

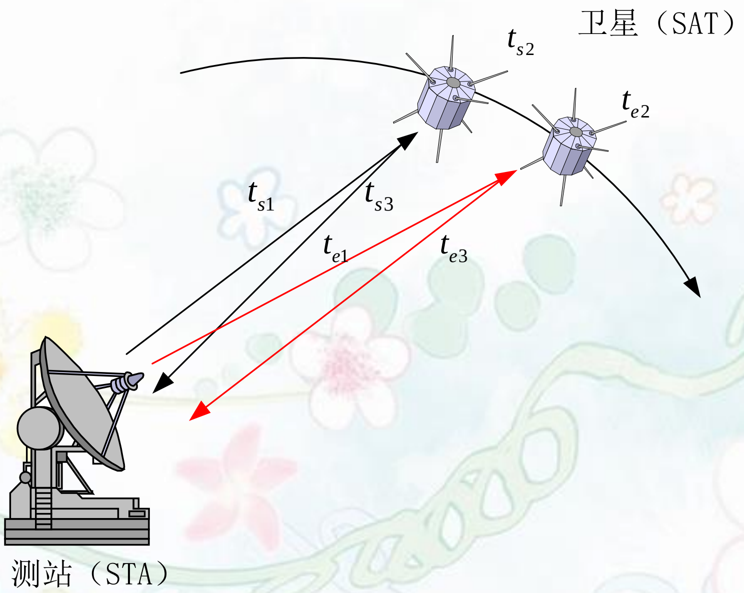
$$\rho_d = \left|\boldsymbol{r}\left(t-\Delta t_2\right) - \boldsymbol{R}\left(t\right)\right| + TR_d + ION_d + GR_d + \varepsilon_d$$

$$\rho_u = \left|\boldsymbol{r}\left(t-\Delta t_2\right) - \boldsymbol{R}\left(t-\Delta t_1-\Delta t_2\right)\right| + TR_u + ION_u + GR_u + \varepsilon_u$$

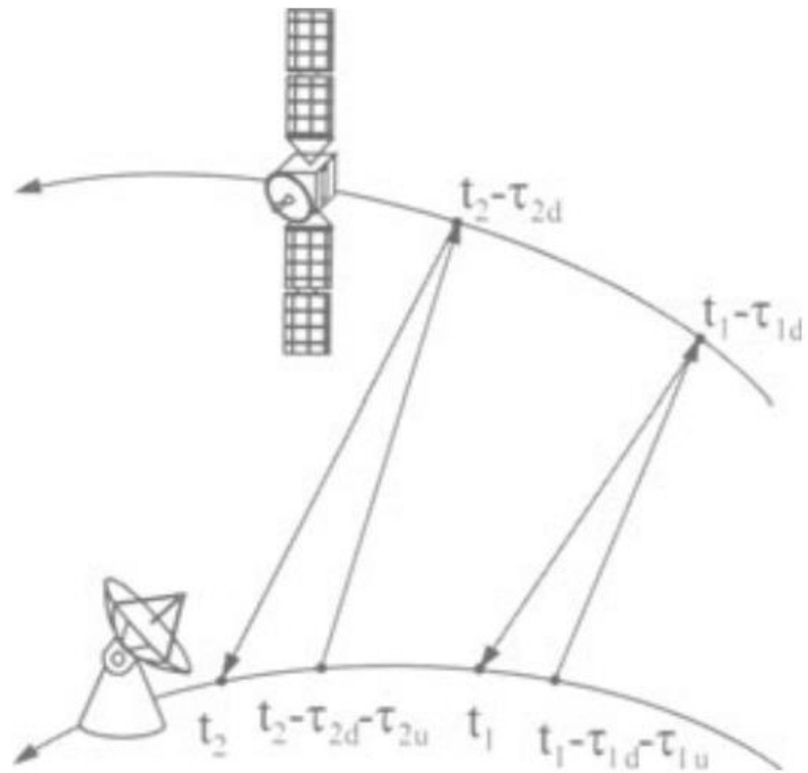
$$\Delta t_2^{\ i+1} = \rho_d\left(\Delta t_2^{\ i}\right) = \frac{1}{c}\left[\left|\boldsymbol{r}\left(t-\Delta t_2^{\ i}\right) - \boldsymbol{R}\left(t\right)\right| + \delta\rho_d\left(\Delta t_2^{\ i}\right)\right]$$

$$\Delta t_1^{\ i+1} = \rho_d\left(\Delta t_2, \Delta t_1^i\right) = \frac{1}{c}\left[\left|\boldsymbol{r}\left(t-\Delta t_2\right) - \boldsymbol{R}\left(t-\Delta t_1^i-\Delta t_2\right)\right| + \delta\rho_u\left(\Delta t_2, \Delta t_1^i\right)\right]$$

多普勒光行时



卫星（SAT）
t_{s2}
t_{e2}
t_{s1}
t_{s3}
t_{e1}
t_{e3}
测站（STA）



$t_2-\tau_{2d}$
$t_1-\tau_{1d}$
t_2
$t_2-\tau_{2d}-\tau_{2u}$
t_1
$t_1-\tau_{1d}-\tau_{1u}$

固体潮对台站坐标影响的计算流程

IERS2010规范中，推荐的固体潮方法是IERS 两步法，即Mathews1996的研究成果，该成果之所以称为两步法。是因为计算固体地球潮汐需要进行两步改正，第一步是时域改正，第二步是频域改正。这是目前国际通用的固体潮汐位移的计算方法，主要原因是时域计算是根据日月坐标进行计算，频域计算只需要考虑比较重要的潮波改正，计算效率明显提升，计算量也比较低。

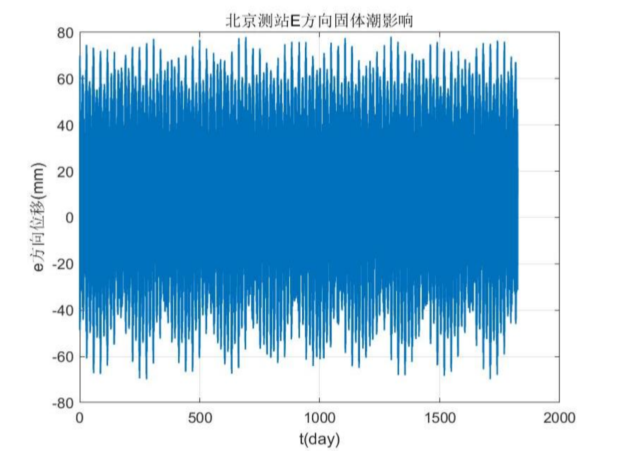
形变公式（二阶与三阶）

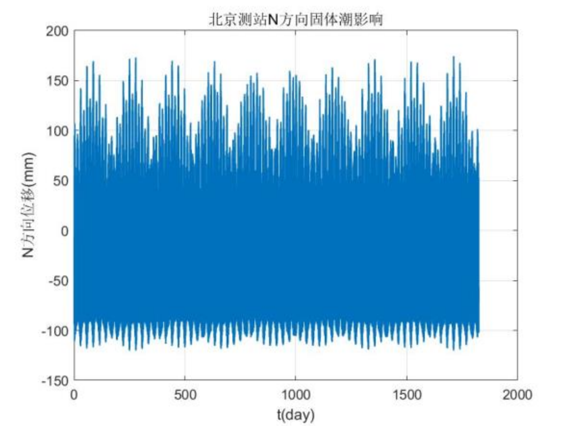
$$\Delta\vec{r} = \sum_{j=2}^{3} \frac{GM_j R_e^4}{GM_\oplus R_j^3} \left\{ h_2\, \hat{r} \left(\frac{3}{2}(\hat{R}_j \cdot \hat{r})^2 - \frac{1}{2} \right) + 3 l_2 (\hat{R}_j \cdot \hat{r}) [\hat{R}_j - (\hat{R}_j \cdot \hat{r})\, \hat{r}] \right\},$$

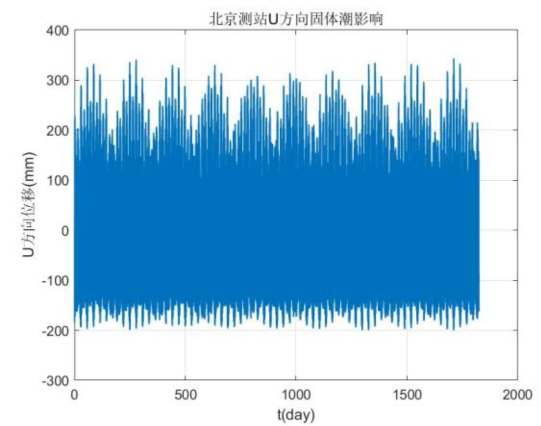
GM_j	=	gravitational parameter for the Moon ($j = 2$) or the Sun ($j = 3$),
$GM_\oplus$	=	gravitational parameter for the Earth,
$\hat{R}_j, R_j$	=	unit vector from the geocenter to Moon or Sun and the magnitude of that vector,
R_e	=	Earth's equatorial radius,
$\hat{r}, r$	=	unit vector from the geocenter to the station and the magnitude of that vector,
h_2	=	nominal degree 2 Love number,
l_2	=	nominal degree 2 Shida number.

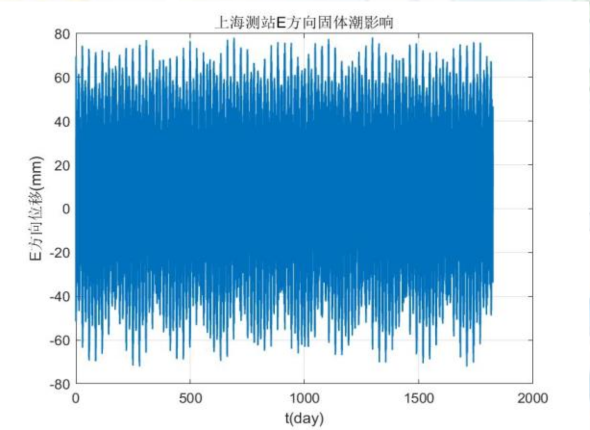
$$\Delta\vec{r} = \sum_{j=2}^{3} \frac{GM_j R_e^5}{GM_\oplus R_j^4} \left\{ h_3\, \hat{r} \left(\frac{5}{2}(\hat{R}_j \cdot \hat{r})^3 - \frac{3}{2}(\hat{R}_j \cdot \hat{r}) \right) + l_3 \left(\frac{15}{2}(\hat{R}_j \cdot \hat{r})^2 - \frac{3}{2} \right) [\hat{R}_j - (\hat{R}_j \cdot \hat{r})\hat{r}] \right\}$$

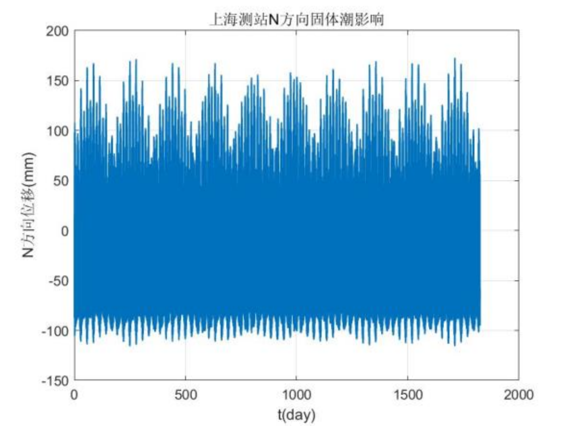
固体潮汐对站坐标影响的时间序列

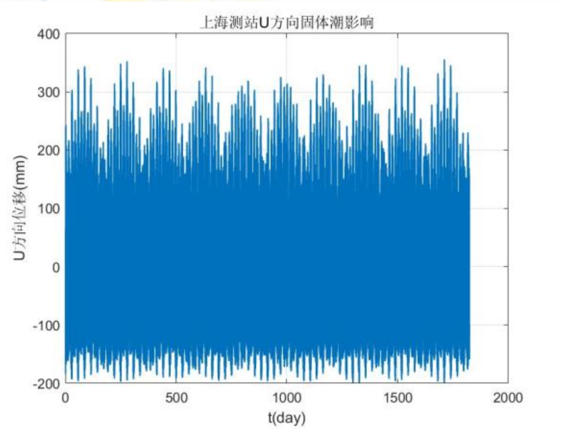




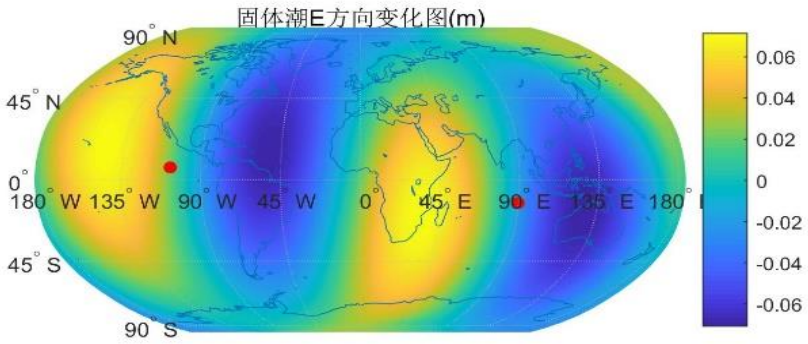


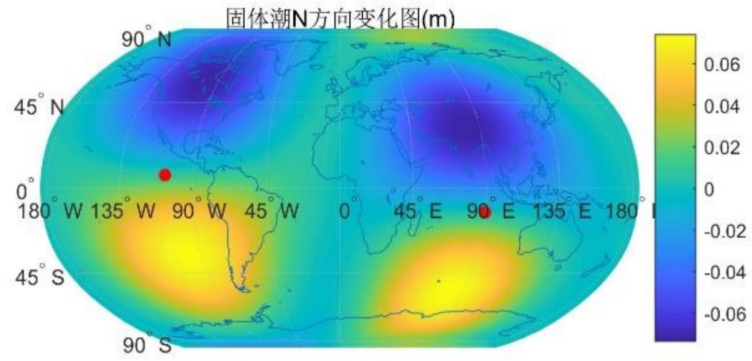


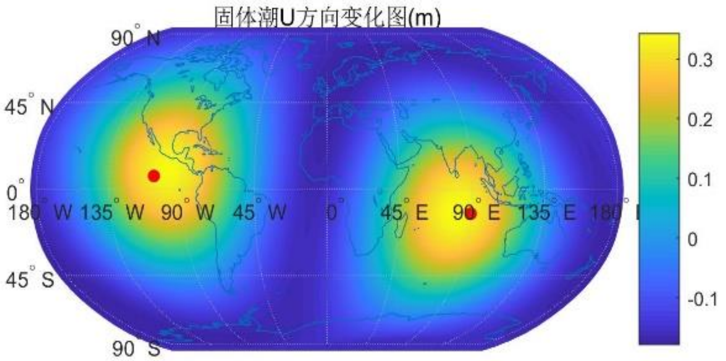




固体潮汐瞬时全球变化量级







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海潮对台站坐标的影响

海潮模型由多项潮波叠加，时间项（第二项）与日月位置有关。振幅项和相位项合称OLC（OCEAN LOADING COEFFCIENT），共6×11项，以IERS2010推荐的BLQ格式存储。

在IERS2010中，推荐的海潮模型是TPXO7.2模型，和FES2004模型。但是在IERS2010中指出，虽然这两个模型是最新的，但是老的模型在某些时候精度比推荐的模型更高。

Model code	Reference	Input	Resolution
Schwiderski	Schwiderski (1980)	Tide gauge	1° × 1°
CSR3.0, CSR4.0	Eanes (1994)	TOPEX/Poseidon altim.	1° × 1°
	Eanes and Bettadpur (1995)	T/P + Le Provost loading	0.5° × 0.5°
TPXO5	Egbert *et al.* (1994)	inverse hydrodyn. solution from T/P altim.	256 × 512
TPXO6.2	Egbert *et al.* (2002), see <[3]>	idem	0.25° × 0.25°
TPXO7.0, TPXO7.1	idem	idem	idem
FES94.1	Le Provost *et al.* (1994)	numerical model	0.5° × 0.5°
FES95.2	Le Provost *et al.* (1998)	num. model + assim. altim.	0.5° × 0.5°
FES98	Lefèvre *et al.* (2000)	num. model + assim. tide gauges	0.25° × 0.25°
FES99	Lefèvre *et al.* (2002)	numerical model + assim. tide gauges and altim.	0.25° × 0.25°
FES2004	Letellier (2004)	numerical model	0.125° × 0.125°
GOT99.2b, GOT00.2	Ray (1999)	T/P	0.5° × 0.5°
GOT4.7	idem	idem	idem
EOT08a	Savcenko *et al.* (2008)	Multi-mission altimetry	0.125° × 0.125°
AG06a	Andersen (2006)	Multi-mission altimetry	0.5° × 0.5°
NAO.99b	Matsumoto *et al.* (2000)	num. + T/P assim.	0.5° × 0.5°

目前常用的海潮模型

IERS推荐不同水域建议的海潮模型

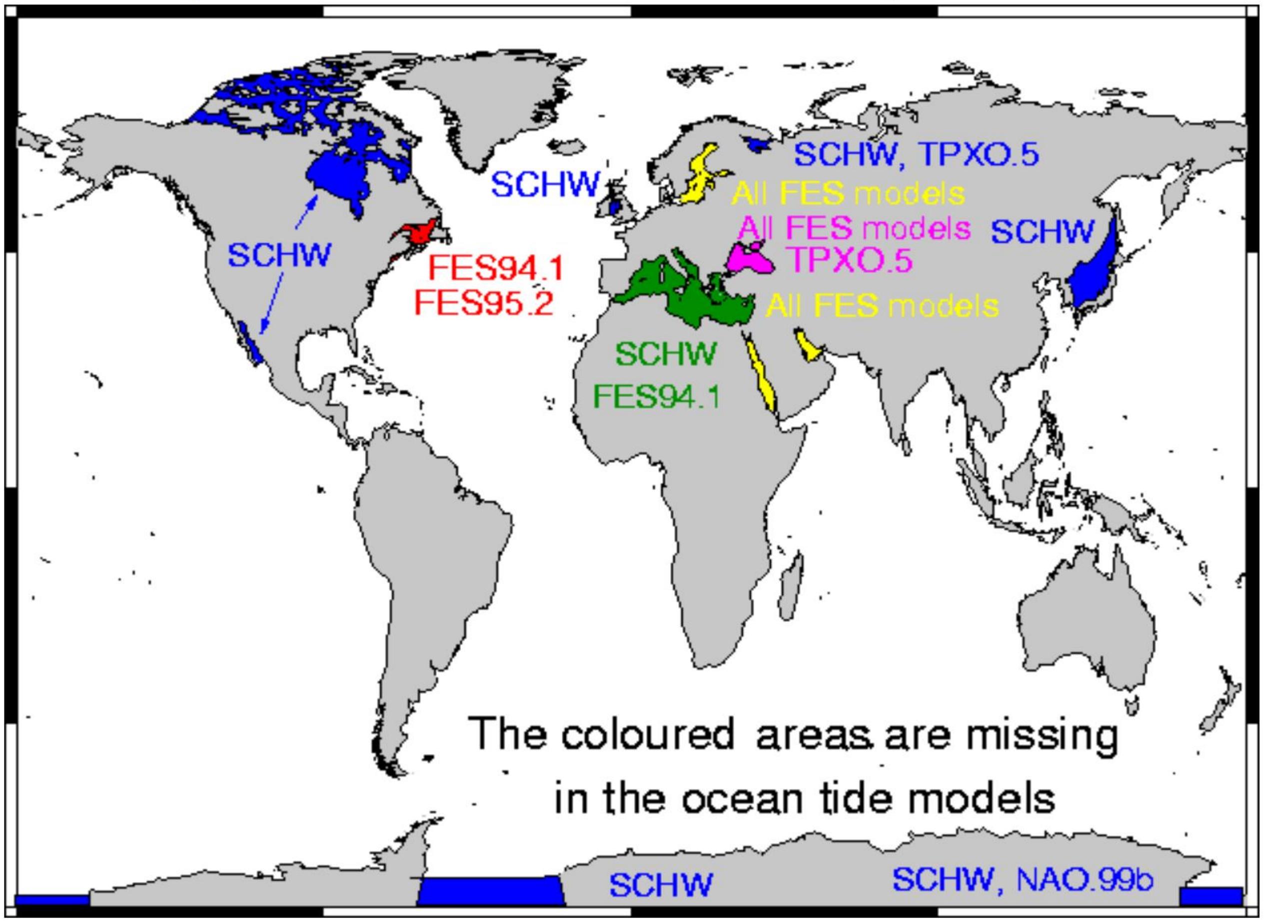


Figure 1: Water areas that are missing in the ocean tide models

极潮对台站的改正

由地球自转产生地球离心力可使地球发生形变，称为极潮，极潮与固体潮一样也可导致地球测站位移（取h_2=0.6090，l_2=0.0852）

$$S_r = -32\sin 2\theta\,(m_1\cos\lambda + m_2\sin\lambda)\ \mathrm{mm},$$
$$S_\theta = -9\cos 2\theta\,(m_1\cos\lambda + m_2\sin\lambda)\ (\mathrm{mm}),$$
$$S_\lambda = 9\cos\theta\,(m_1\sin\lambda - m_2\cos\lambda)\ \mathrm{mm},$$

其中x_P、y_P为极移，θ、λ为测站余纬和经度。

$$[dX, dY, dZ]^T = R^T[S_\theta, S_\lambda, S_r]^T$$

$$R = \begin{pmatrix} \cos\theta\cos\lambda & \cos\theta\sin\lambda & -\sin\theta \\ -\sin\lambda & \cos\lambda & 0 \\ \sin\theta\cos\lambda & \sin\theta\sin\lambda & \cos\theta \end{pmatrix}$$

固体潮、海潮与极潮的改正

$$\Delta\rho_{TD} = \left\{ \Delta\vec{r}^{\,sta} + (MLT)^T \begin{pmatrix} \delta E_P^{sta} + \delta E_{OT}^{sta} \\ \delta N^{sta} + \delta N_P^{sta} + \delta N_{OT}^{sta} \\ \delta h_1^{sta} + \delta h_2^{sta} + \delta h_P^{sta} + \delta h_{OT}^{sta} \end{pmatrix} \right\} \times \frac{\vec{r} - \vec{r}^{\,sta}}{\left| \vec{r} - \vec{r}^{\,sta} \right|}$$

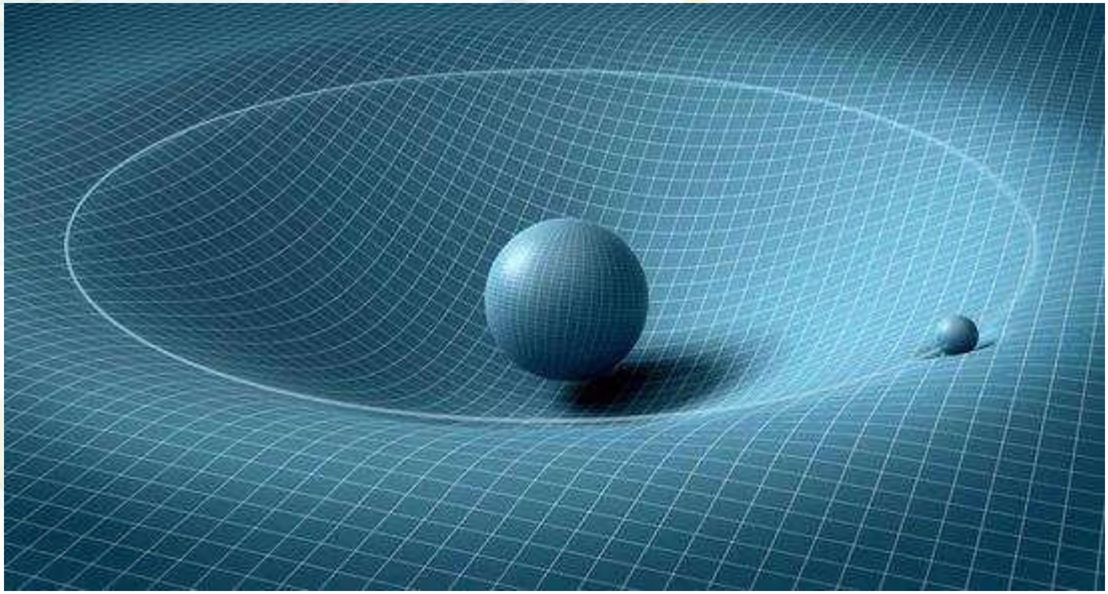
$(MLT)^T$是站心坐标转换到地固坐标的旋转矩阵

$$(MLT) = \begin{pmatrix} -\sin\lambda & \cos\lambda & 0 \\ -\cos\theta\cos\lambda & -\cos\theta\sin\lambda & \sin\theta \\ \sin\theta\cos\lambda & \sin\theta\sin\lambda & \cos\theta \end{pmatrix}$$

相对论引力时延

在平直空间中光的传播速度是不变的数值C。当存在引力场时，光的坐标速度不再是常数，而是恒小于C的变量，这使光在引力场中传播的时间比在平直空间中传播的时间要长，其差额就是引力场造成的，称为引力时延，它不是爱因斯坦揭示的相对论效应，是由夏皮罗（I.I.Shapiro）于1964年提出的，因而又称为夏皮罗时延，记为Δt_G

$$\Delta t_G = \frac{2GM}{C^3} \ln \frac{r + R + \rho}{r + R - \rho}$$



观测量相关偏导数（测距为例）

$$G(x,t)=[(\vec{R}-\vec{R}^{sta})^T(\vec{R}-\vec{R}^{sta})]^{1/2}$$
$$=[(\vec{r}-\vec{r}^{sta})^T(\vec{r}-\vec{r}^{sta})]^{1/2}=\rho$$

$$\tilde{H}_i=\left(\frac{\partial G}{\partial X}\right)_i^*$$

$$X=\left[\vec{R}^T \,\vdots\, \vec{V}^T \,\vdots\, C_D, C_R, x_{p0}, \dot{x}_p, y_{p0}, \dot{y}_p, D_R\right]^T$$

$$\frac{\partial G}{\partial \vec{R}}=\frac{(\vec{R}-\vec{R}^{sta})^T}{\rho}$$

$$\frac{\partial G}{\partial \dot{\vec{R}}}=0$$

$$\frac{\partial G}{\partial \vec{p}_d}=0 \qquad \vec{p}_d=[C_D,C_R]^T$$


Q&A!