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Perturbing Effects of a Close Star on Relativistic Precessions of a Star in Galactic Center

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Abstract: As proposed by Clifford M. Will, observations of the precessing orbits of a class of stars around the supermassive black hole in the galactic center with very short period ($O(0.1)$ yr) could provide measurements of the spin and quadrupole moment of the black hole and thereby test the no-hair theorems of general relativity, if the observation accuracy from the Earth can be reached 10 μas and 1 μas in the near future. However, in the Galactic center, many researches showed that there is a stellar density cusp, a region of diverging density around the supermassive black hole. The result is that the observed stars are perturbed by other stars unavoidably. In order to investigate the influences of such perturbing effects, we numerically calculate the perturbation effects on precessions of perihelion and orbital plane coming from a very close perturbing star and then compare with the precessions caused by the Schwarzschild part (mass), frame-dragging effect (angular momentum) and quadrupole moment. Our results show that the perturbing effect is much smaller than the Schwarzschild precession, but possibly can affect the measurement of the frame-dragging effect and quadrupole moment, especially for the latter one.

Key words: black hole physics; Galaxy: center; relativity

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1 Introduction

It is widely believed that there is a supermassive black hole (SMBH) associated with the radio source Sgr A* in our Galactic center beyond any reasonable doubt[1–3]. The mass of supermassive black hole was estimated about 4 million solar mass [4, 5]. The distance from the SMBH to the Sun is about 8 kpc, is 100 times closer than the SMBH in Andromeda, the nearest large galaxy. For this reason, the Galactic black hole offers the best laboratory for strong gravitational field physics and testing general relativity (see reviews[6, 7] for more details).

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Using stars near the Galactic SMBH can detect post-Newtonian effects and test general relativity (GR) in the weak and strong field limits near the SMBH. Several successful experiments have done in our solar system for detecting very weak GR effects [8–11], but the SMBH will give us the best chance to test GR in strong gravitational field near a supermassive black hole. The S2 star, one of a group of S stars at distances ranging from $10^0 - 10^2$ mpc from the Galactic center, with an orbital period about 15 years, has a periapse advance about 0.2°/yr based on general relativity. But the Lense-Thirring effect of S2 is too small to be detected now. As suggested by Will[12], if we can find some stars around the Galactic center at very small semimajors $a \lesssim 1$ mpc and with high eccentricity, the orbital plane precessions induced by the spin angular momentum $\boldsymbol{J}$ and quadrupole moment Q of the SMBH can be larger than 10 μas/yr observing from the Earth and can be detected by some coming projects, for example, GRAVITY. Further more, observed precessions in orbital planes of two such stars can in principle fix $\boldsymbol{J}$ and Q of the SMBH then test general relativistic "no-hair" theorems which demands that $Q = -J^2/M$ (where M is the mass of the SMBH).

These measurements require that the observed sources are in area which the distance less than 1 mpc from the Galactic center. In such very central region, one can not just take the observed stars as test mass but need consider potential complications. Numerous theoretical model with different dynamical initial and boundary conditions showed that the distribution of stars around the SMBH forms a stellar density cusp, a region of diverging density around the SMBH[13–17]. This means in $\lesssim 1$ mpc region, there are dense of stars including stellar black holes. And extrapolating the observed stellar densities at distances of 1 pc from the SMBH show that of order $10^0 - 10^2$ stars should be in this region [18]. Accordingly, the orbit of target star definitely can be perturbed by other stars nearby. Thus perturbation from another star maybe can pollute the measurements of these relativistic precessions if the perturbing star is massive and closes enough.

Merritt et al. (2010) did a N-body simulation and discussed the influence of star distribution on the measurements of general relativistic precessions[18]. And in 2011, Sadeghian and Will discussed the almost same problem by analytic orbital perturbation theory[19]. They concluded that for a range of possible stellar distributions, the effects of stellar cluster perturbations can not obscure the relativistic spin and quadrupole effects for an observed star sufficiently close to the black hole. These results are consistent with those from the numerical N-body simulations by [18].

However, both of the above two researches focused on the perturbation effect of N-star distribution in the surrounding cluster (each star is far away from the target one) on the target star's orbital precessions and comparing with the relativistic effects. As a different one from these two researches, in the present paper, we consider the perturbation effect comes from only one perturbing star but is very close to the target star but not a moon of it. We simulate this three-body system numerically and find that the perturbation from such close star is possible to pollute the observations of relativistic precessions especially the quadrupole one. Consequently we must state here that the focus of our work is different from the two papers[18, 19] though they look similar. And the results are also different because we think that the perturbation effect can interfere the measurements of the relativistic effects.

The paper is organized as follows. In Sec. II we briefly introduce the relativistic precessions of a star near the SMBH in the Galactic center. And we describe the dynamical equations of our three-body system, and give a simple analysis based on the orbital perturbation theory. Then in Sec. IV, we present our numerical results. In the last section, conclusions and discussions are made.

2 Relativistic effects in Galactic center and dynamical equations

As mentioned in Sec. I, a supermassive black hole with $M = 4\times10^6 M_\odot$ locates in our Galactic center. The Schwarzschild radii of the SMBH is about 0.08 AU and 10 μas see from the Earth. If a star with semimajor axis a orbits around the SMBH, the orbital period in unit of year is:

$$P = \frac{2\pi a^{3/2}}{\sqrt{GM}} \approx 1.48\tilde{a}^{3/2}, \tag{1}$$

where $\tilde{a}$ means the semimajor axis in units of mpc. From Eq. (1), we can see if a star with semimajor axis $0.1 \sim 1$ mpc, the period is about $0.1 \sim 1$ yr.

The orbital periapse and plane precessions per orbit are given as [12],

$$\Delta\bar{\omega} = A_{\rm S} - 2A_J\cos i - \frac{1}{2}A_Q(1-3\cos^2 i), \tag{2}$$

$$\Delta\Omega = A_J - A_Q\cos i, \tag{3}$$

where $\Delta\bar{\omega} = \Delta\omega + \cos i\Delta\Omega$ is the precession of pericenter relative to the fixed reference direction, i, ω and Ω are the orbital inclination, argument of periapse, and longitude of ascending node respectively. $A_{\rm S}$, A_J and A_Q represent the relativistic effects due to the black hole's mass (Schwarzschild part), angular momentum (frame-dragging) and quadrupole moment respectively [19] in unit of arcmin/yr, they can be written as:

$$A_{\rm S} = \frac{6\pi}{c^2}\frac{GM}{(1-e^2)a} \approx 8.351(1-e^2)^{-1}\tilde{a}^{-5/2}\,, \tag{4}$$

$$A_J = \frac{4\pi\chi}{c^3}\left[\frac{GM}{(1-e^2)a}\right]^{3/2} \approx 0.0769(1-e^2)^{-3/2}\chi\tilde{a}^{-3}\,, \tag{5}$$

$$A_Q = \frac{3\pi\chi^2}{c^4}\left[\frac{GM}{(1-e^2)a}\right]^{2} \approx 7.979\times10^{-4}(1-e^2)^{-2}\chi^2\tilde{a}^{-7/2}\,. \tag{6}$$

where c is the velocity of light, e the orbital eccentricity, and $\chi = J/M^2$ the Kerr parameter. We can easily find that the Schwarzschild precession is much larger than the other two.

For the simplicity and focusing on the main purpose, we here adopt a Newtonian gravitational equations but not post-Newtonian one to calculate the perturbation effects. Because that the relativistic precessions have been analytical worked out (see Eqs. (2-6)), and gravitational field of the perturbing star is weak enough to use Newtonian gravity. The omission of the post-Newtonian terms in our numerical simulations does not change the results in this paper, because in our simulations we want to figure out the third body's perturbation. The post-Newtonian terms added or no added in the numerical codes will be no influence on the extraction of the perturbation effect from the near star on the target one. Considering the mass of the SMBH is much larger than the orbiting stars, the acceleration of target star can be written as,

$$\boldsymbol{a} = -\frac{GM\boldsymbol{R}}{R^3} - \frac{Gm_*\boldsymbol{r}}{r^3}, \tag{7}$$

where m_* is the mass of perturbing star, $\boldsymbol{R}$ is the vector from the SMBH pointing to the target star and $\boldsymbol{r}$ the vector from the perturbing star to the target one. r is very small comparing to R, but is large enough to avoid the two stars becoming a binary. It is very easy to estimate the Hill radius of the target star with circular orbit, $r_H \approx a(m/3M)^{1/3} \approx 4.4 \times 10^{-3}a$. For high eccentricity, the Hill radius will be smaller. Thus r must be larger than r_H.

Though the distance between the target and perturbing stars is very small, the mass of the perturbing star is only 10^{-6} of the SMBH. We still can consider the gravitational force of m_* as a perturbation on the orbit of the target star.

Based on a simple perturbation analysis, we can find that for the target star, because of the perturbation, there is no secular variation of the semimajor a, but the inclination i and eccentricity e have slowly and secularly contrary changes due to the Kozai mechanism[20], and also the orbital periapse and plane exist secular precessions. As we know, even with the numerical simulations, completely investigating the precession by the third body perturbation in qualitative and quantitative is quite difficult, because the perturbation magnitude depends on too many parameters. The parameters we chose in our numerical models in the next chapter try to make the perturbation relatively large in order to study its influence on the possible measurements of the relativistic effects in the future.

3　Numerical results

Based on the analysis in the last chapter, the semimajor of the target star is set from 0.1 mpc to about 1 mpc, and the difference of semimajors between the target star and the perturbing one is 0.01 mpc. The last number is based on the extrapolation estimation which suggests that there are of order $10^0 - 10^2$ stars inside this region. And also, this separation makes that the two stars are very hard to enter the Hill sphere of each other. It is means that they are impossible to become a binary in the background gravitational field of the SMBH. The mass of target star is one solar mass and the mass of perturbing one can be set as one, two or three solar mass. For simplification, we put the initial orbit of the perturbing star on the equatorial plane of the SMBH, but the target one have an orbital inclination i (the angle between the orbital angular momentum of the star and the spin of the SMBH). And we set the Kerr spin parameter as its maximum value 1 during all calculations.

For calculating the values of precessions numerically, we simulate the orbital evolutions about a few ten thousand periods to make the precession rates ($\dot{\Omega}$) tend to constant values. Then we take these constants as the average values of precession rates (in unit arcmin/yr). A Runge-Kutta-Fehlberg 7(8) integrator is adopted for integrating the dynamical equations. And we check the conservations of energy and angular momentum to make sure the numerical precision and reliability.

In the top-left panel of Fig. 1, we plot the orbital precessions of a high eccentricity star ($e = 0.9$) produced by the spin and quadrupole of the SMBH, and the perturbation caused by a one solar-mass nearby star. It can be clearly seen that the perturbation effect is hard to be observed by the instruments with 10 μas precision in the whole region we considered. At the same time, the frame-dragging effect can be observed if the semimajor $a \lesssim 0.7$ mpc and the quadrupole one also can be observed if the star is closer to the Galactic center $a \lesssim 0.17$ mpc. But for a 1 μas observation accuracy, the perturbation effect completely covers the quadrupole part for 0.2 mpc $\lesssim \tilde{a} \lesssim$ 0.5 mpc.

Now, we keep all parameters except the mass of perturbing star being changed to $m_* = 3M_\odot$, we can see the quadrupole effect will be polluted obviously. Even the frame-dragging effect will be covered by the perturbation for $\tilde{a} \gtrsim 0.7$ mpc with 1 μas observation accuracy (see the top-right panel of Fig. 1).

Then, in the bottom-left panel of Fig. 1, we put the perturbing star with one solar mass a little far from the target star, as $|\tilde{a}_* - \tilde{a}| = 0.1$ mpc. We can find that for the 10 μas observation, we can not see the perturbation effect. For the 1 μas accuracy, the perturbation has a small influence on the quadrupole effect observation, but almost nothing on the frame-dragging observation.

Furthermore, the magnitude of perturbation also depends on the eccentricity e and orbital inclination i. Based on our numerous simulations, usually, a higher eccentricity and smaller inclination will produce a larger perturbation precession. We use $e = 0.95$, $i = 0.3$ to replace the values of eccentricity and inclination in the mentioned panels (0.9 and 0.1 respectively). The results are shown in the bottom-right panel. Though having a higher eccentricity, because of the bigger orbital inclination ($i = 0.3$), the perturbation effect almost has no influence on the frame-dragging and quadrupole observations with both the 10 μas and 1 μas precision.

In addition, with the future 10 μas even 1 μas observation technology, people can measure the Schwarzschild precessions of the S-stars which have been observed, for example, the

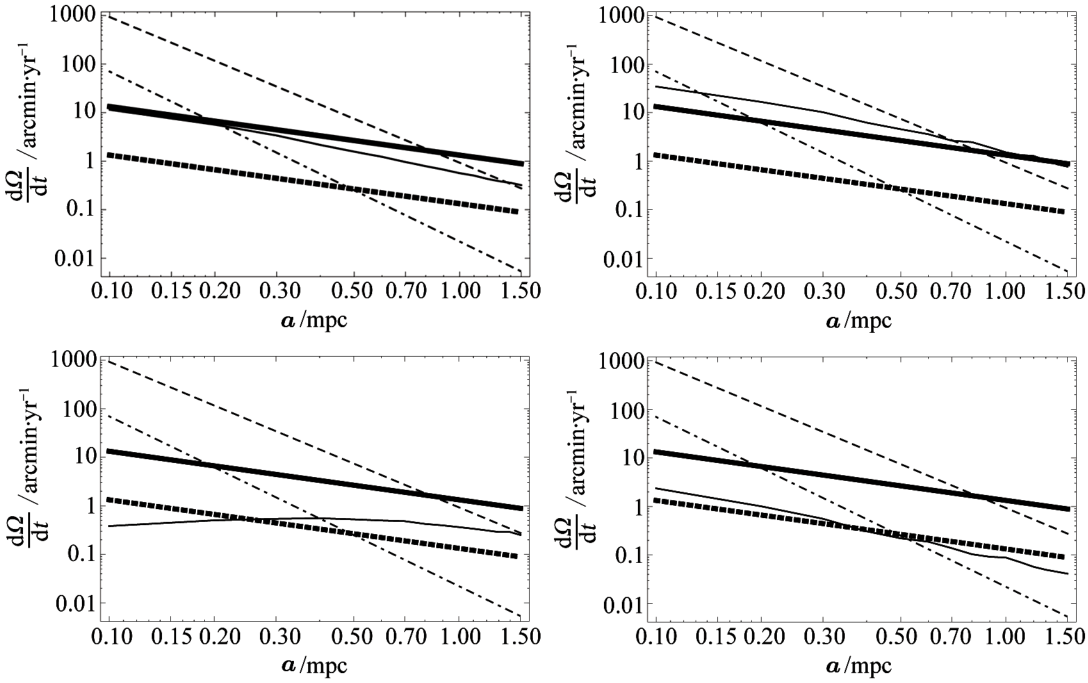

Fig. 1　Comparison of the third body perturbation effect with the SMBH's spin and quadupole effects for different orbital parameters. The dashed curve, the dot-dashed and the solid one represent the frame-dragging, quadrupole and perturbation effects respectively; The thick solid and thick dashed curves are the 10 μas/yr and 1 μas/yr observation levels from the Earth respectively (hereafter the same). The orbital parameters: $m_* = m = 1\ M_\odot$, $\Omega_* = \Omega$, $\omega_* = \omega$, $e_* = e = 0.9$, $i = 0.1$ and $|\tilde{a}_* - \tilde{a}| = 0.01$ mpc (the "*" labels the corresponding variables of the perturbing star)(top-left); $m_* = 3m = 3\ M_\odot$, $\Omega_* = \Omega$, $\omega_* = \omega$, $e_* = e = 0.9$, $i = 0.1$ and $|\tilde{a}_* - \tilde{a}| = 0.01$ mpc (top-right); $m_* = m = 1\ M_\odot$, $\Omega_* = \Omega$, $\omega_* = \omega$, $e_* = e = 0.9$, $i = 0.1$ and $|\tilde{a}_* - \tilde{a}| = 0.1$ mpc (bottom left); $m_* = m = 1\ M_\odot$, $\Omega_* = \Omega$, $\omega_* = \omega$, $e_* = e = 0.95$, $i = 0.3$ and $|\tilde{a}_* - \tilde{a}| = 0.01$ mpc (bottom right).

S2 star with the orbital period about 15 years (the shortest period in all observed S-stars). A problem is that it needs tens years observations to determine the values of precessions because of the long orbital period. Just for a demonstration, we plot here the perturbation effect on the Schwarzschild precession (induced by the mass part of the SMBH). We can see clearly that the perturbation cannot influence on the observations for the target stars with semimajor from 5 to 50 mpc (see Fig. 2).

4　Conclusions and discussions

It is quite difficult to completely describe the third body's perturbation effect on the target star, because the perturbation magnitude depends on too many orbital parameters: mass, semimajor, eccentricity, inclination, and so on. In this paper, we try to use some relatively large perturbations to investigate the influence of the precession induced by the perturbation on the relativistic precessions. Our numerical simulations show that the pre-

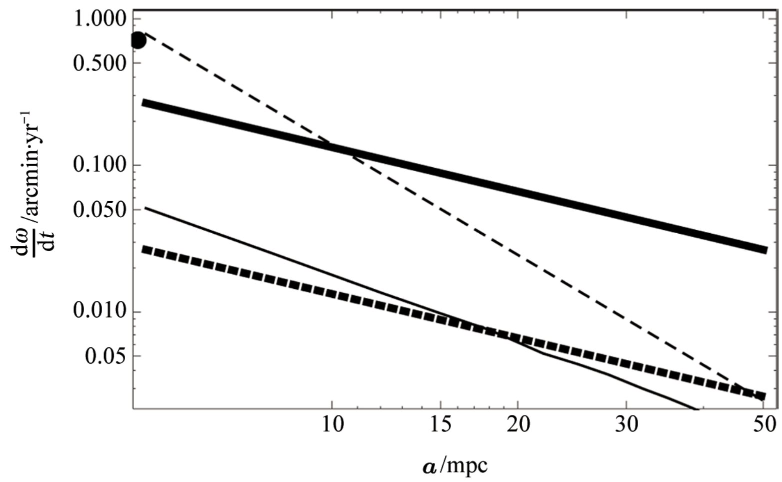

Fig. 2 Comparison of the third body's perturbation effect with the SMBH's Einstein precession (the mass part, represented by the dashed curve). The orbital parameters are: $m_* = m = 1\ M_\odot$, $\Omega_* = \Omega$, $\omega_* = \omega$, $e_* = e = 0.9$, $i_* = 0.1$ and $|\tilde{a}_* - \tilde{a}| = 0.1$ mpc.The black point is the corresponding position of the S2 star.

cession observations of the very near stars ($a \sim O(0.1)$ mpc) around the central black hole in our Galaxy to test the frame-dragging and quadrupole effects predicted by general relativity can not exclude the possibility of being polluted by the third body's perturbation.

However, it is also possible that there are more than one perturbing star unseen around the target star in this very central region, and this situation will make the problem much more complicated. And then, of course, it must can influence the measurement accuracy of the frame-dragging and quadrupole moment. We need to do some N-body simulations including at least 1 PN (post-Newtonian) terms to investigate this problem in details. In addition, 10 μas and even 1 μas observation level can make us measure the Schwarzschild precessions (induced by the mass part of the SMBH) of the observed S-stars, though it needs about tens years observation data. As an example, we also numerically calculate the perturbing effect on thus upcoming observation, and the results show that the perturbing star has no influence on the observation in our model.

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银河系中心区域近星摄动对相对论进动的影响

韩　文　标

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摘　要： Clifford M. Will 提出，通过观测以很短周期 ($O(0.1)$ 年) 围绕银河系中央超大质量黑洞旋转的一组恒星的轨道进动，在未来的 10 μas 甚至 1 μas 微角秒的观测精度下 (从地球)，能够测量中央黑洞的自旋和质量四极矩，从而能够检验广义相对论中的黑洞无毛定理。但是，许多研究表明，在星系中央存在一个围绕中央超大质量黑洞的恒星密度极高区域。这导致观测目标星的轨道运动会不可避免地受到其他星体的引力摄动影响。为了调查这些摄动的影响，本文数值计算了一个非常靠近观测目标的星体对目标星体的引力摄动产生的近心点和轨道平面进动，并把这些进动的大小和相对论进动中的施瓦西部分 (质量引起)、参考架拖曳效应 (黑洞自旋引起) 以及四极矩部分分别作了比较。结果发现，摄动效应对施瓦西进动几乎不会产业影响，但是可能会影响参考架拖曳效应和黑洞四极矩 (特别是后者) 的观测。

关　键　词： 黑洞物理；星系中心；相对论