

# 银河系的厚盘成分和现代恒星计数分析

容建湘

(南京大学天文系 南京 210093)

## 摘 要

在本文中介绍有关银河系成分的认识及星族概念的新进展,对近年新发现的一个恒星成分“厚盘”的主要特性以及利用现代恒星计数分析测定厚盘大尺度结构参数的方法作了较详细的论述。

**关键词** 银河系:中心—银河系:晕—银河系:恒星容量—银河系:结构—球状星团:一般

## The Thick Disk of the Galaxy and Analysis of Modern Starcounts

Rong Jianxiang

(Department of Astronomy, Nanjing University, Nanjing 210093)

## Abstract

This paper reviews the new development of the conception about stellar populations and what the major components of the Milky Way are. Also, it discusses in detail the main characteristics of the thick disk, a most recently recognized stellar component of our Galaxy, and how to determine the structural parameters of the thick disk by analysing the data of modern starcounts.

**Key words** Galaxy:center—Galaxy: halo—Galaxy: stellar content—Galaxy: structure—globular: general

# 1 Development of the Conception of Stellar Populations

P. ten Bruggencate first brought out the conception of stellar populations in 1927 in his book "Sternhaufen". He divided the stars into two groups, population I and population II, according to the differences in HR diagrams between Galactic and globular clusters. In 1944, Baade observed the central parts of M 31 and M 33, and found that the HR diagrams of bright stars were quite similar to that of globular clusters in the Galaxy, while the HR diagrams for the bright stars in the outer parts of those galaxies were close to that of Galactic clusters. Based on his observations, Baade also presented the conception of two basic stellar populations. The terminology of stellar populations is widely used ever since<sup>[1]</sup>.

According to Baade, the stars in our and other spiral galaxies may be divided into two groups, population I and population II, and there are many differences in the shape of HR diagrams, color and luminosity of brightest stars between them. The brightest population I stars are white early supergiants, whereas the brightest population II stars are red K supergiants. In spatial distributions and motions, the two populations are different, too. The population I stars have strong concentration towards the Galactic plane and they are restricted to a thin disk, but few of the stars are at the center. The population II stars have strong central concentration and are distributed in a spheroid or oblate spheroid with the center of the galaxy as its center. The population I stars move rapidly around the Galactic center in an orbit quite close to a circle, and the plane of orbit differs from the Galactic plane only by a small angle. On the contrary, the population II stars move around the Galactic center in an elliptical orbit with large eccentricity, and usually the moving velocity is small, however the dispersion of velocities is large, and the angle between the orbit plane and the Galactic plane is large. Stars of population I are relatively younger than those of population II. Generally speaking, population II stars, globular clusters and the Galaxy itself have the same order of magnitude of age.

A conference on stellar populations held at Vatican in 1957<sup>[2]</sup>. The participants agreed that it was oversimplified to divide stars into only two groups. Later, Oort outlined a scheme of five population types (extreme population I, intermediate population I, disk population, intermediate population II, and halo population II) and made them a continuous sequence to describe star features such as spectra, luminosities, chemical abundances, ages, spatial and kinematic properties, etc<sup>[3]</sup>. This classification has played a dominant role in the studies of galactic components for many years<sup>[4]</sup>.

After the conception of stellar populations prevailed for half a century, however, it seems to become outmoded and to be abandoned gradually mainly because it caused some confusions. In fact, at the Vatican conference, five stellar populations were for spanning the range between Baade's two extremes. But this classification has proven unequal to

the more recent recognition that there are young stars of low metal abundance in the Magellanic Clouds, in other galaxies, and even in the outer parts of the Galaxy, So, quite a few astronomers would rather return to simple descriptors-mainly age (or age range) and abundance than use the traditional terminology of stellar populations<sup>[5]</sup>.

## 2 The Modern Knowledge of the Components of the Milky Way

What are the major components of the Milky Way? The most obvious to us is the thin disk, which contains most of the stars of our immediate neighborhood. Its distribution is approximately exponential both in the radial direction and in  $z$ , with scale heights of a few kiloparsecs and a few hundred parsecs, respectively. The density law is

$$D(x, z) = n_0 e^{-(x-R_0)/d} e^{-z/h} \quad (1)$$

with scale length  $d$  and scale height  $h$ , where  $x$  and  $z$  are the Galactrocentric cylindrical coordinates of a given point (distance from the Galactic center, projected upon the Galactic plane, and height above the Galactic plane, respectively);  $R_0$  is the distance of the Sun from the Galactic center;  $n_0$  is the local density of the thin disk. Stars in the thin disk move in orbits around the Galactic center that differ only little from circles. The abundances are very close to those of the Sun, although there is almost certainly a modest radial gradient toward lower abundances with increasing distance from the center. Ages of stars, and of open clusters, range from nearly that of the Milky Way down to practically zero<sup>[5-8]</sup>.

Further researches show that the thin disk can be divided into a young thin disk and an old thin disk, for which the density laws are also expressed by Eq. (1), but with different local density and especially different scale height.

Also well known is the stellar halo, consisting of the globular clusters and about 100 times as many field stars. The globular clusters, at least, can be divided into two subgroups, the so-called halo globulars and disk globulars. The halo globulars have a nearly round distribution that is approximated by a radial  $-3.5$ -power law; their system is pressure-supported and has little or no rotation around the Galactic center. The disk globulars, on the other hand, have a flatter spatial distribution and a mean rotation around the Galactic center that is about half the circular speed. The metal abundances of the halo globulars are very low, and those of disk globulars are moderately low. The globular clusters are the oldest objects in the Milky Way. Whether there are age differences among them (particularly between the halo group and the disk group) is controversial.

The status of the field stars of the halo is in many ways puzzling. On the whole, they seem to be the counterparts of the halo globular clusters, but some of the correspon-

dences are not as close as they might be. And the counterpart of the disk globulars is even more puzzling. It is tempting, but speculative and in no way compelling, to identify them with the thick disk that is described below.

The density function for the halo component is expressed by the deprojected de Vaucouleurs  $r^{1/4}$  law:

$$D(s) = n_0 \exp \left[ -f r_e^{-1/4} \left( S^{1/4} - R_0^{1/4} \right) \right] (S/R_0)^{-7/8} \quad (2)$$

with the argument  $s$  being defined as

$$s^2 = x^2 + z^2/q^2 \quad (3)$$

and

$$x = \left( R_0^2 + r^2 \cos^2 b - 2R_0 r \cos b \cos l \right)^{1/2} \quad (4)$$

$$z = r \sin b \quad (5)$$

where  $q$  is an axial ratio of the equidensity surface and  $0.6 \leq q \leq 1.0$ ;  $f=7.669$  is a numerical constant;  $R_0$  is the distance of the Sun from the Galactic center;  $r_e$  is the effective radius of the halo and nearly equal to  $R_0/3$ ;  $n_0$  is the local density of the halo;  $r$  is the distance from the Sun and  $l, b$  are the Galactic coordinates. Eq. (2) can be expressed approximately by

$$D(s) = n_0 (s/R_0)^{-7/8} e^{10.093[1-(s/R_0)^{1/4}]} \quad (6)$$

In the central part of the disk is the bulge—perhaps a part of the disk, perhaps a separate component, but certainly not merely the center of halo, from which it differs drastically in metal abundance. The bulge is, moreover, difficult to observe; and insofar as we observe it, it is a mixture. Metal abundances of the old stars range from low to far above solar. The bulge is dominated by old stars, but we do not know if they are all equally old. Its flattening is modest, with comparable amounts of rotational and pressure support.

There is also another stellar component called thick disk in the Galaxy, which we will discuss in the next paragraph.

Except above stellar components, there are two non-stellar components in the Milky Way. The interstellar medium, most of which is thinner even than the thin disk, is the key to present and future star formation. And the dark halo, known only by its gravitational effect, is still very much a mystery. There are indications that it is rather round, and almost certainly not made of stars<sup>[9]</sup>.

### 3 The Thick Disk—a Most Recently Recognized Stellar Component

The Galactic thick disk is the intermediate population II by the terminology from the 1957 Vatican Conference on stellar populations.

Some edge-on galaxies with bulges show a second flattened component, the thick disk, in addition to the usual thin disk. In 1979, Burstein *et al.* investigated the detailed luminosity structure of five edge-on galaxies. The fainter parts of the perpendicular profiles in all five galaxies require the presence of a third luminosity component, in addition to a thin disk and a bulge, termed as a “thick disk”. As evidenced from the photometric data on those galaxies, a thick disk has the following general properties: (1) it is of intermediate flattening between a bulge ( $z/a \approx 0.5$ ) and a thin disk ( $z/a < 0.1$ ); (2) it is more diffuse than a thin disk, with an exponential luminosity gradient (of scale height about 1 kpc) perpendicular to the major axis; (3) it has a very shallow luminosity gradient parallel to the major axis; and (4) it has a distinctive rectangular-boxed shape in edge-on galaxies at faint surface-brightness levels<sup>[10–12]</sup>.

Star counts by Gilmore and Reid (1983), confirmed by Yoshii *et al.* (1987), showed that the Galaxy also has a thick disk, with scale height of about 1 kpc, column density of an order of 10 percent of that of the thin disk, and local normalization to the thin disk of about 0.02, and the density law also has the same form as Eq.(1)<sup>[13–15]</sup>.

The thick disk raises some interesting questions about its origin and implications for galaxy formation:

Are thick disks found only in galaxies with significant central bulges? This apparently well defined observational question does not yet have a definitive answer, because it is difficult to disentangle the luminosity distributions of the bulge/spheroidal component from that of the thick disk. We do know that some disk galaxies do not have thick disk, so thick disk formation is not an essential part of the formation and evolution of disk galaxies<sup>[16]</sup>.

Are thick disks in rotational equilibrium like thin disks, or are they rotating more slowly (as if they were an intermediate population formed during the collapse)?

How old are the stars of the thick disk? Did the thick disk form very early in the life of the parent galaxy (like the metal-weak halo) or later (like the thin disk)?

At present, it is not possible to study the kinematics and chemical properties of the thick disks in other galaxies; it is still difficult to establish their existence. To understand the properties of the thick disks, we must rely on detailed observations of kinematics, chemical abundances and ages of the thick disk stars in the Galaxy<sup>[17–24]</sup>.

The kinematical properties of the galactic thick disk can be measured from observations of stars with metallicities between about  $-0.6$  and  $-1.0$ . For these stars, the vertical

velocity dispersion is about  $40 \text{ km} \cdot \text{s}^{-1}$  and their scale height is therefore about 1 kpc. The radial component of the velocity dispersion is about  $65 \text{ km} \cdot \text{s}^{-1}$  and the asymmetric drift is about  $30 \text{ km} \cdot \text{s}^{-1}$  relative to the LSR. The thick disk is clearly a rapidly rotating population<sup>[25–31]</sup>.

Most recent work indicates that the thick disk near the Sun is very old, i.e. at least as old as the disk globular clusters<sup>[32–36]</sup>. Nissen and Schuster (1991) have used uvby photometry to show that the turn-offs of the local thick disk and halo stars have similar ages<sup>[37]</sup>. In 1993, Carney studied the distributions of dereddened  $(B-V)_0$  colors for metal-poor stars, thick disk-metallicity stars and thick disk-kinematics stars respectively, and the distribution of orbital eccentricities against the periods for thick disk spectroscopic binaries. Carney's results support the above claim<sup>[18]</sup>. So, if the white dwarf luminosity function is a reliable indicator of the age of the classical thin disk, we must conclude that star formation was delayed for a very long time, several billion years, between the star formation eras of the halo and the thick disk, and the star formation era of the thin disk.

Many theories for the origin of the Galactic thick disk have been reviewed by Gilmore *et al.*<sup>[38–40]</sup>. Essentially the formation models fall into “top-down” scenarios, where the formation of the thick disk precedes that of the thin disk, and “bottom-up” scenarios whereby the thick disk is formed through some action on or by the thin disk. In general, the “top-down” scenarios view the thick disk as a dissipational structure formed as a transitional phase between the formation of the halo and the formation of the thin disk. As a dissipational entity, the thick disk is expected to exhibit evidence of “spin-up”—the increase in rotational velocity due to conservation of angular momentum as the thick disk gas collapsed. Thus, kinematic gradients are expected as a function of distance above the Galactic plane. Whether vertical metallicity gradients also develop depends on the relative rates of collapse and enrichment<sup>[41]</sup>.

The “bottom-up” scenarios take on several forms. It has been popular to suggest the accretion of satellites into the disk as an explanation for the existence of thick disk component in galaxies<sup>[42]</sup>. The most thorough examination of the problem of satellite accretion and their effects on galactic disk has been the N-body simulation work of Quinn *et al.*<sup>[43,44]</sup>.

Thus the two most favored and competing models for the formation of the thick disk are the dissipational and the accretion-induced heated disk models. These models may be distinguished once the properties of the thick disk are firmly established. For example, a strong vertical thick disk kinematic gradient would favor the dissipational models, whereas a modest or nonexistent thick disk kinematic gradient would favor the accretion model. Unfortunately, the kinematics of the thick disk are not firmly established.<sup>[18,40,45]</sup>

In recent years the determination of structural parameters for the thick disk has become interesting to astronomers. It is a part of analysis of modern starcounts<sup>[46]</sup>.

## 4 The Structural Parameters for the Thick Disk and Analysis of Modern Starcounts

### 4.1 Modern starcounts

The most extensive set of starcount data is that for the galactic poles -BVI observations covering 18 square degrees towards the south pole; the Kiso Schmidt data, covering 25 square degrees towards the north pole; the BV data over a similar area in the NGP from Palomar Oschin Schmidt data; and the extensive SA 57 dataset. And the UBV data for SA 54 are provided by Yamagata *et al.*

The Basel RGU three-color photographic high-latitude survey provides starcount data in 15 fields distributed systematically near the meridional plane of the Galaxy. For the past three decades, their halo survey has been one of the major systematic studies of the large-scale distributions of the stellar populations in the Milky Way.

The starcounts can be denoted by  $A(V, B-V)$ , which is the number of stars in a unit solid angle, whose apparent magnitude  $V$  lies between  $V - 1/2$  and  $V + 1/2$  and color index  $B-V$  lies between  $B-V-0.05$  and  $B-V+0.05$ .

### 4.2 The fundamental integral equation of stellar statistics

In order to compare the observed data with the theoretical models, the following fundamental equation of stellar statistics can be used:

$$N(V, B - V) = \sum_{i=1}^k \int_0^{\infty} D_i(r) \varphi_i[M_V, (B - V)_0] r^2 dr \quad (7)$$

where  $N(V, B-V)$  is the number of stars in a unit solid angle, whose apparent magnitude  $V$  lies between  $V - 1/2$  and  $V + 1/2$  and color index lies between  $B - V - 0.05$  and  $B - V + 0.05$ ;  $D_i(r)$  is the stellar space-density distribution for  $i$ -th stellar component, whose distance from the Sun is  $r$ ;  $\varphi_i[M_V, (B - V)_0]$  is the luminosity function for  $i$ -th stellar component, which is a function of absolute magnitude  $M_V$  and intrinsic color index  $(B - V)_0$ ; and

$$M_V = V + 5 - 5 \log r - A(V, r) \quad (8)$$

$$(B - V)_0 = B - V + A(V, r) - A(B, r) \quad (9)$$

where  $A(V, r)$  and  $A(B, r)$  are the total extinction for magnitude  $V$  and  $B$  respectively, which depends on the distance  $r$ . In Eq. (7) the subscript  $i$  distinguishes the functions for different components. For example, if  $K = 2$ , it refers to the disk and halo components; if  $K = 3$ , it refers to the thin disk, thick disk and halo components; if  $K = 4$ , the thin disk is divided into a young thin disk and an old thin disk. Eq. (7) is for UBV photometric system. For other photometric systems one can get similar equations.

Thus, to predict starcounts in a given direction, one requires a luminosity function and density distribution for each component in the model. For density distribution of each component, the reader can refer to Eqs (1) and (2).

It is difficult to determine luminosity functions because they are different from each other for different components, and even for the same component the luminosity function for giants is different from that for dwarfs. For the thin disk, Wielen's luminosity function from the Gliese nearby star catalog is often adopted<sup>[47-49]</sup>. For the thick disk and halo, the luminosity functions are derived by using observed data for globular clusters. To establish the luminosity functions relative to intrinsic color indexes, the B-V and U-B two-color relations for main-sequence stars and giant-sequence stars have to be used. It has been known that the two-color relations for main sequence and giant sequence are different and they both depend on metallicities. So, the final luminosity functions, which are two-dimensional and relative to luminosity and color index, are for a given component (thin disk, or thick disk, or halo), a given stellar sequence (main or giant sequence) and a given metallicity. If metal abundance depends on space position, the luminosity functions are relative to space.

For the corrections of interstellar extinction, we have on average

$$A(V, r) = R_V E_{(B-V)} = a_V \cdot r \quad (10)$$

$$A(B, r) = R_B E_{(B-V)} = a_B \cdot r \quad (11)$$

where  $R_V$  and  $R_B$  are the extinction ratios for V and B magnitudes respectively;  $E_{(B-V)}$  is the color excess;  $a_V$  and  $a_B$  are the extinction values in unit length for V and B magnitudes respectively. The detailed researches on extinction have been done by Buser<sup>[50-53]</sup>.

### 4.3 Star count analysis and thick disk parameters

After giving definite values to all parameters in models, the  $N(V, B-V)$  (and  $N(V, U-B)$ ) predicted by the theoretical models can be calculated by Eq. (7). Because we don't know the exact parameter values, we first set up boundaries for some parameters, and calculate the  $N(V, B-V)$  and  $N(V, U-B)$  for the given parameter values. Then we compare the calculated results with starcount data  $A(V, B-V)$  and  $A(V, U-B)$ . In this manner, we can choose the most reliable value for each parameter. If there are several parameters and we have several values to be selected for each parameter, there will be a large number of possible models. For this reason, the calculation load is very heavy.

In two-dimensional starcounts  $A(V, B-V)$ , the interval of V is 1mag or 0.5mag and the interval of B-V is 0.1mag or 0.2mag. Thus, the observed data are for a lot of bins in the V and B-V domain. If the available bin numbers for  $A(V, B-V)$  and  $A(V, U-B)$  are  $N_1$  and  $N_2$  respectively, the residual square sum between the starcount data



and the result predicted by a model is

$$\chi^2 = \sum_{i=1}^{N_1} (A_i(V, B - V) - N_i(V, B - V))^2 + \sum_{j=1}^{N_2} (A_j(V, U - B) - N_j(V, U - B))^2 \quad (12)$$

Computing a number of models iteratively until  $\chi^2$  is minimized on the domain of the parameters, the corresponding parameter values are the best expected. It is evident that, in the least square solution, the bins are weighted nearly proportional to the number of stars located in them.

To simulate the observed starcounts, a modification is applied to the model distribution of  $N(V, B - V)$  and  $N(V, U - B)$  to take account of the measurement errors. A Gaussian function with different dispersions is used to  $B - V$  and  $U - B$ .

The above is an introduction of the analysis of modern starcounts provided by the computer algorithms in the last ten years for modeling the stellar populations and distributions in the Galaxy. Besides, the traditional way to study the Galactic components by separating the different luminosity groups among the population I (disk) and II (halo) has been given by Fenkart *et al.*, and its most recent application is Fenkart's synoptic study of the Basel high-latitude data, where the empirical densities are compared with those predicted by a standard set of five multi-component models for the Galactic space density distributions<sup>[54-58]</sup>.

The analysis of modern starcounts is the best way for determining the model parameters of Galactic populations in large-scale structure. It has provided a strong evidence of the presence of a thick disk component in the Galaxy, whose scale height is about 1 kpc, scale length is about 4 kpc, and mean metallicity is about  $-0.76$  dex. Whether the metallicities diminish with the increasing of the distances from the Galactic plane or not still remains unanswered. From the analysis of starcounts, the local density of the thick disk is 1 percent or several percent of the thin disk density<sup>[13-15,59]</sup>, but using some other independent methods Sandage obtained a higher result which is about 10% of the local thin disk<sup>[60-63]</sup>.

## References

- [1] Baade W. *Ap. J.*, 1944, 100: 137
- [2] O'Connell D J K ed. *Stellar populations*. Amsterdam: North Holland, 1958
- [3] Oort J H. In: O'Connell D J K ed. *Stellar population*. Amsterdam. North Holland, 1958: 419
- [4] 容建湘. *恒星天文学*. 北京: 高等教育出版社, 1986. 220
- [5] Gilmore G *et al.* *The Milky Way as a galaxy*. Geneva: SAAS-FEE. 1989. 3
- [6] Gilmore G, Carswell B. *The galaxy*. Dordrecht: Reidel, 1987. 118
- [7] Gilmore G *et al.* *Annu. Rev. Astron. Astrophys.*, 1989, 27: 555
- [8] Majewski S R. In: Majewski S R ed. *Galaxy evolution: The Milky Way perspective*, ASP conference series 49, San Francisco: ASP, 1993: 5
- [9] Little B, Tremaine S. *Ap. J.*, 1987, 320: 493

- [10] Tsikoudi V. *Ap. J.*, 1979, 234: 824
- [11] Burstein D. *Ap. J.*, 1979, 234: 829
- [12] Tsikoudi V. *Ap. J. Suppl.*, 1980, 43: 365
- [13] Gilmore G, Reid N. *M. R. A. S.*, 1983, 202: 1025
- [14] Yoshii Y *et al.* *A. J.*, 1987, 93: 323
- [15] Yoshii Y. *Publ. Astron. Soc. Jpn.*, 1982, 34: 356
- [16] van der Kruit P C, Searle L. *Astron. Astrophys.*, 1981, 95: 105
- [17] Majewski S R. In: Majewski S R ed. *Galaxy evolution: The Milky Way perspective*, ASP conference series 49, San Francisco: ASP, 1993: 17
- [18] Carney B W. In: Majewski S R ed. *Galaxy evolution: The Milky Way perspective*, ASP conference series 49, San Francisco: ASP, 1993: 83
- [19] Freeman K C. In: Majewski S R ed. *Galaxy evolution: The Milky Way perspective*, ASP conference series 49, San Francisco: ASP, 1993: 125
- [20] Norris J E, Ryan S G. *Ap. J.*, 1991, 380: 403
- [21] Berman B G, Suchkov A A. *Astrophysics Space Sci.*, 1991, 184: 169
- [22] Berkert A *et al.* *Ap. J.*, 1992, 391: 651
- [23] Katz N. *Ap. J.*, 1992, 391: 502
- [24] Nissen P E, Schuster W J. *Astron. Astrophys.*, 1991, 251: 457
- [25] Sandage A, Fouts G. A. J., 1987, 92: 74
- [26] Laird J *et al.* *A. J.*, 1988, 95: 1843
- [27] Norris J E. *Ap. J.*, 1987, 314: L39
- [28] Yoss K *et al.* *A. J.*, 1987, 94: 1600
- [29] Fried E D. *A. J.*, 1988, 95: 1727
- [30] Ratnatunga K, Freeman K C. *Ap. J.*, 1989, 339: 126
- [31] Armandroff T. A. J., 1989, 97: 375
- [32] Nissen P. In: Terlevich R ed. *Elements and the cosmos*. Cambridge: Cambridge Univ. Press, 1990. 41
- [33] Rose J, Agostinho R. A. J., 1991, 101: 950
- [34] Carney B *et al.* *A. J.*, 1989, 97: 423
- [35] Norris J E, Green E M. *Ap. J.*, 1989, 337: 272
- [36] Norris J E. In: Brodie J P, Smith G H eds. *The globular cluster-galaxy connection*, ASP conference series 48, San Francisco: ASP, 1992: 3
- [37] Nissen P E, Schuster W J. *Astron. Astrophys.*, 1991, 251: 457
- [38] Freeman K C. In: Majewski S R ed. *Galaxy evolution: The Milky Way perspective*, ASP conference series 49, San Francisco: ASP, 1993: 125
- [39] Gilmore G *et al.* In: *The Milky Way as a galaxy*. Geneva: SAAS-FEE, 1989. 227
- [40] Majewski S R. *Annu. Rev. Astron. Astrophys.*, 1993, 31: 71
- [41] Sandage A. *Roy. Astron. Soc. Can.* 1990, 84: 70
- [42] Reid I N, Majewski S R. *Ap. J.*, 1993
- [43] Quinn P J *et al.* *Ap. J.*, 1992, 403: 74
- [44] Hernquist L, Quinn P. In: Majewski S R ed. *Galaxy evolution: The Milky Way perspective*, ASP conference series 49, 1993: 187
- [45] Majewski S R. *Ap. J., Suppl.*, 1992, 78: 87
- [46] Reid N. In: Majewski S R ed. *Galaxy evolution: The Milky Way perspective*, ASP conference series 49, San Francisco: ASP, 1993: 37
- [47] Gliese W. *Veröff. Astron. Rechen Inst. Heidelberg No. 22*, 1969

- [48] Gliese W. *Astron. Astrophys. Suppl.*, 1982, 47: 471
- [49] Wielen R *et al.* In: Davis Philip A G, Upgren A R eds. *The nearby stars and the stellar luminosity function*, Proc. of IAU colloq. No. 76. Middletown, USA, 1983, New York: L. Davis Pris Press, 1983: 163
- [50] Buser R, Fenkart R P. *Astron. Astrophys.*, 1990, 239: 243
- [51] Buser R *et al.* *Astron. Astrophys.*, in press
- [52] Buser R. *Astron. Astrophys.*, 1978, 62: 411
- [53] Buser R. *Astron. Astrophys.*, 1978, 62: 425
- [54] Fenkart R P. *Astron. Astrophys. Suppl.*, 1989, 78: 217
- [55] Fenkavt R P. *Astron. Astrophys. Suppl.*, 1989, 79: 51
- [56] Fenkart R P. *Astron. Astrophys. Suppl.*, 1989, 80: 89
- [57] Fenkart R P. *Astron. Astrophys. Suppl.*, 1989, 81: 187
- [58] Fenkart R P, Karaali S. *Astron. Astrophys. Suppl.*, 1990, 83: 481
- [59] Yamagata T, Yoshii Y. *A. J.*, 1992, 103: 117
- [60] Sandage A. A. J., 1987, 93: 610
- [61] Sandage A. A. J., 1987, 93: 592
- [62] Natie J S. Roy. *Astron. Soc. Can.*, 1990, 84: 175
- [63] Buser R *et al.* *Astron. Astrophys.*, in press